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APPROXIMATE REANALYSIS IN TOPOLOGY OPTIMIZATION

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ABSTRACT

Despite rapid improvements in computer performance there is still room for advances in computational efficiency for handling structural optimization applications due to the high computational cost involved in the optimization process. In general, the total computational cost of the optimization procedure is governed by the complexity of three aspects: the model, the analysis and the optimization [1]. Solving problems that consider high complexities in all three aspects is presently somewhat limited in problem size, thus motivating the search for methods and procedures that require reduced computational resources but yield high quality results.

In this paper, the implementation of an approximate reanalysis method in topology optimization is investigated. We apply the so-called nested approach where optimization is performed in the design variables only and where the equilibrium equations are treated as function calls. The goal is to reduce the computation involved in repeated solutions of the equilibrium equations, which for large problems will dominate the computational cost of the whole process. The approximate reanalysis performed here is based on the Combined Approximations (CA) approach, proposed by Kirsch [2] for linear static reanalysis and later used successfully also in vibration, dynamic and nonlinear reanalysis. The main feature of CA is the integration of a local series expansion in a global reduced basis solution. When using CA for repeated structural analysis, one can substantially reduce the number of required factorizations of the stiffness matrix, and thus removing a significant portion of the computational effort.

In this study, CA has been implemented for topology optimization of linear structures, and the accuracy of the solutions obtained has been benchmarked against solutions presented in the literature ([3] and [4]). The relative efficiency of the method has been evaluated by comparing with the case in which all analysis equations are solved by a direct method. There is a clear trade-off between the accuracy and the efficiency of the approximate procedure. Both are mainly influenced by two parameters: the frequency of full matrix decompositions and the number of vectors generated for the reduced basis. More frequent decompositions will result in smaller changes in stiffness and therefore a more accurate approximation, but at the cost of matrix factorizations; similarly, using more basis vectors will give a better representation of the actual solution but will increase the computational effort. For all the trial cases considered, the optimal designs and the objective values obtained by the approximate procedure were practically

identical to those obtained by the full procedure and the relative errors were negligible. Moreover, the number of iterations required for the optimization process to converge did not vary significantly when using the approximate method, meaning that the accuracy achieved is indeed sufficient.

The potential savings in computational effort depend on the chosen solution scheme. Several solution schemes are defined, aimed at providing a good balance between the frequency of matrix factorizations and the number of basis vectors generated. The computational experiments show that the number of factorizations can be reduced to 7%-15% of the original number of factorizations, depending on the specific scheme used. When considering problems with a large number of degrees of freedom, a theoretical speedup factor can be evaluated by an operations count for each procedure and speedup factors of 4 for 10^5 DOF (FEM degrees of freedom), 8.5 for 10^6 DOF and 14.5 for 10^7 DOF can be achieved; this order of speedup is verified by experiments. Higher savings can be achieved by introducing better control of the solution parameters - the frequency of the decompositions and the number of basis vectors considered - in accordance to the actual changes in the design variables and objective function. As the optimization proceeds towards convergence, the design changes are smaller so less frequent decompositions and fewer basis vectors are required. This issue will be examined thoroughly in the presentation.

When the number of solution vectors that are required at each optimization iteration increases, for example when we need to solve several adjoint problems for sensitivity analysis or when multiple loadcases are considered, the relative efficiency of the approximate procedure decreases. This is due to the fact that the basis vectors depend on the right hand side (load) vector in the equation system, meaning that for each load vector a different basis should be generated. When solving by a direct method, once the stiffness matrix was decomposed then obtaining solution vectors for multiple load vectors is practically for free. Usually, only a few constraints are considered in topology optimization (in most current implementations at least), so the number of adjoint equations is typically not too large. When considering, for example, two adjoint equations, we reach speedup factors of 4.5 for 10^6 DOF and 8.5 for 10^7 DOF; when considering five adjoint equations, the speedup factors are reduced to 2.5 for 10^6 DOF and 5.5 for 10^7 DOF.

This work illustrates that approximate reanalysis in general and the CA approach in particular could be attractive when trying to reduce the computational effort involved in large scale topology optimization. When many adjoint equations or load vectors are considered, the relative efficiency decreases. However, in topology optimization problems that require non-linear analysis at each iterative step, an approximation based on CA could be efficient, since the number of adjoint equations will be much smaller than the number of repeated analyses.

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