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A BDDC PRECONDITIONER FOR THE REISSNER-MINDLIN PLATE BENDING PROBLEM

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ABSTRACT

The Reissner-Mindlin model is the most widely used linear model for the description of the bending problem for elastic isotropic plates. In general engineering analysis the preferred finite element family for this problem is the so called MITC, i.e. Mixed Interpolation by Tensorial Components Finite Elements, since this method is free of locking together with a set of other useful properties [2]. The related discrete problem for a plate of midsurface Ω reads

$$\begin{aligned}&\text{Find } (\boldsymbol{\theta}, w) \in \mathbf{X}_h \text{ s.t.}\\ &a(\boldsymbol{\theta},\boldsymbol{\eta})_{L^2(\Omega)} + k\, t^{-2}(\nabla w - \Pi_h\boldsymbol{\theta}, \nabla v - \Pi_h\boldsymbol{\eta})_{L^2(\Omega)} =< g, v > \quad \forall(\boldsymbol{\eta}, v) \in \mathbf{X}_h\end{aligned} \tag{1}$$

where $\boldsymbol{\theta}, w$ stand respectively for the rotations and deflections, $\mathbf{X}_h$ represents the MITC discrete space including boundary conditions, a is a continuous and coercive bilinear form on rotations, Π_h is the shear reduction operator, g is the normal scaled loading, $k = 5/6$ the shear correction factor and t is the plate thickness.

Differently from the simpler shear-free Kirchhoff model, the thickness t enters into the problem as a "penalty" parameter. As a consequence, there is a large detrimental effect on the condition number of the respective linear problem which, in the thickness, behaves roughly as t^{-2}. Since this is a model for thin structures, the relative thickness is in general in the $t \sim 10^{-2}/10^{-4}$ range, which implies a $\sim t^4/t^8$ multiplicative factor in the condition number. Such thickness effect combines with the usual condition number explosion in h, i.e. as the mesh size tends to zero, giving a possibly very ill-conditioned system.

The aim of this contribution is to introduce a Domain Decomposition preconditioner for the Reissner-Mindlin plate bending problem which guarantees a condition number bound independent of t and of the number of subdomains (scalability). To the authors knowledge this is the first result in the DD preconditioning of the very popular MITC Reissner-Mindlin plate elements.

The contribution is organized as follows. After a brief introduction to the MITC Reissner-Mindlin problem and related numerical difficulties, a BDDC (Balancing Domain Decomposition by Constraints) preconditioner is presented. Afterwards, the t-uniform scalability of the resulting linear system is shown; although the proof has its roots in the works [3,4], new specific techniques for the Reissner-Mindlin plate problem have been developed. Finally, we show a set of numerical tests which agree with the theory and underline the robustness of the condition number of the preconditioned system. The subject of this talk has been submitted for publication [1].

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