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NUMERICAL METHOD FOR THE PROBLEM BENDING VIBRATION OF A THIN ELASTIC PLATE AND ITS APPLICATION IN THE PROBLEM OF A FLOATING ICE VIBRATION

***A. Kuleshov, V. Mymrin**

Institute for Mathematical Modeling
of Russian Academy of Sciences
4-A, Miusskaya sq., Moscow, Russia, 125047
E-mail: andrew_kuleshov@mail.ru

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ABSTRACT

The problem of small bending vibrations occurring in a thin elastic plate of variable thickness with a bending moment and a shearing force applied to the plate contour is considered. A difference approximation to the original equation based on a three-level difference scheme is inconvenient and has a multipoint stencil, whose numerical implementation is difficult. In this work, when constructing a difference approximation of the problem, we replace the original equation, which is of the fourth order with respect to spatial variables and of the second order with respect to time, by a system of equations with first-order time derivatives. A two-level implicit scheme is considered for solving this system. For numerical solution of this system the two-layer implicit scheme is created. This scheme is more convenient for the numerical solution of the problem, than the three-layer scheme.

The equation of elastic vibrations of a thin isotropic plate of variable thickness lying on an elastic (Winkler) foundation has the form [1]

$$\rho h \frac{\partial^2 W}{\partial t^2} + \Delta\left(D\Delta W\right) - (1-\sigma)\left(\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 W}{\partial x^2} - 2\frac{\partial^2 D}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y} + \frac{\partial^2 D}{\partial x^2}\frac{\partial^2 W}{\partial y^2}\right) + aW = F, \qquad (1)$$

where $W(x,y,t)$ is the plate deflections which is measured along the z axis, ρ is the density of the plate material, $D = Eh^3/[12(1-\sigma^2)]$ is the cylindrical stiffness of the plate, E is the modulus of elasticity, σ is Poisson's ratio of the plate material, F is the external force given on a surface of a plate.

On the curvilinear contour of a plate general (Kirchhoff-generalized) conditions: the bending moment and the vertical shearing force are given. The initial conditions:

$$W\big|_{t=0} = \varphi(x,y), \qquad W_t\big|_{t=0} = \psi(x,y).$$

Let us use the well-known approach to reducing a high-order partial differential equation to a system of equations of smaller order. In the equation of plate vibrations, we make the substitution $S = W_t$. Also we differentiate on t formulas for the bending moments on x and on y. Then instead of the equation (1) we receive system of the equations of the first order on time:

$$\rho h \frac{\partial S}{\partial t} = \frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - aW + F,$$

$$\frac{\partial M_x}{\partial t} = -D\frac{\partial^2 S}{\partial x^2} - \sigma D\frac{\partial^2 S}{\partial y^2}, \qquad (2)$$

$$\frac{\partial M_y}{\partial t} = -\sigma D\frac{\partial^2 S}{\partial x^2} - D\frac{\partial^2 S}{\partial y^2},$$

$$\frac{\partial W}{\partial t} = S,$$

where M_x, M_y are bending moments, M_{xy} is a torsion moment:

$$M_x = -D\left(\frac{\partial^2 W}{\partial x^2} + \sigma\frac{\partial^2 W}{\partial y^2}\right), \quad M_y = -D\left(\frac{\partial^2 W}{\partial y^2} + \sigma\frac{\partial^2 W}{\partial x^2}\right), \quad M_{xy} = (1-\sigma)D\frac{\partial^2 W}{\partial x \partial y}.$$

Boundary conditions for the bending moment and the vertical shearing force are given. The initial conditions:

$$W\big|_{t=0} = \varphi, \quad S\big|_{t=0} = \psi, \quad M_x\big|_{t=0} = -D(\varphi_{xx} + \sigma\varphi_{yy}), \quad M_y\big|_{t=0} = -D(\varphi_{yy} + \sigma\varphi_{xx}).$$

For numerical solution of the system (2) the two-layer implicit scheme is created. Difference approximation of the system is under construction on a rectangular mash [2].

The system of difference equations was solved by splitting with respect to variables *x* and *y* into two subsystems. To solve each of these subsystems efficiently, we renumber the unknowns in a certain order and reduce the system of linear algebraic equations to a system with a nine-diagonal matrix, which is solved by Gaussian elimination. In this case, the coefficients are calculated in four stages. Test calculations have shown the high efficiency of the method proposed for solving the problem.

The developed method has been applied to a problem of floating ice elastic vibrations under action of dynamic loadings. Results of numerical simulation of wave propagation in a floating ice under action one and several moving objects are represented. At simultaneous movement of several objects one after another, depending on distances between them, it was observed either clearing of waves, or their imposing with increase in amplitude.

REFERENCES

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