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Application of Dimension Reduction Methods to the Dynamics of a Fluid Conveying Tube

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ABSTRACT

Realistically modelled mechanical systems usually are represented in an infinite dimensional phase space creating great difficulties in analyzing their dynamics. Hence one will try to reduce the original system to a low dimensional system. However this is useful only if a good approximation of the original dynamics is achieved.

In this presentation we consider dimension reduction methods for the simulation of the discretized equations for a fluid conveying tube. Due to the presence of viscous internal damping in the equations of motion the spectrum of the linear operator has a finite accumulation point, which strongly influences the applicability of Approximate Inertial Manifolds: Even if a considerable number of eigenfunctions is chosen as dominating modes, the long term behaviour of the full system and the reduced system differ considerably. The same difficulty can also be observed in a very simple system with two degrees of freedom.

In order to improve the convergence behaviour, we consider different sets of basis functions for the approximation of the dominant modes:

Reordered eigenfunctions Since due to the viscuous damping some modes with spatially small frequency have a large negative eigenvalue, we include these modes in the dominating set.

Beam modes The leading eigenfunctions of a conservative cantilevered beam should approximate the governing dynamics quite well. Since we calculate these modes from the conservative system, the first few mode shapes should already provide a reasonable approximation for the dominant states.

POD modes By extracting the "permanent states" from the results of previous simulations, one might hope to find the best possible choice for the dominating modes, at least for nearby parameter values.

Wavelet basis In our numerical simulations we frequently observed, that simulations with a coarse discretization gave better results than sophisticated reduction methods with a fine mesh. The coarse discretization could therefore provide the dominating basis.

Classically the dimension reduction by the Ritz-Galerkin method simply neglects the influence of the "slave modes" on the system dynamics. In some situations this could give completely wrong answers. Several methods have been proposed to take the influence of the slave modes into account: For systems close to a bifurcation boundary the local behaviour can be reduced by the *Center Manifold* reduction, which gives the correct answer, but is very cumbersome to carry out, expecially if higher orders are needed.

The different kinds of *Approximate Inertial Manifolds*, which ignore the elaborate terms in the Center Manifold calculations, are much easier to obtain and have been applied very successfully in a large range of examples. In these cases a gap condition for the spectrum was satisfied, which guaranteed, that the high order modes decay sufficiently fast. For the fluid conveying tube problem the accumulation point in the spectrum prevents the fast decay of the high order modes. Using a simple 2-mode mechanical model we demonstrate, that for the correct calculation of the long term behaviour the Center Manifold approach gives the right terms, whereas the Approximate Inertial Manifold reduction fails to take care of the energy transfer to the slave mode.

By separating the dominant and slave dynamics already in the implicit second order system, instead of converting the discrete ODEs to first order equations and then applying the reduction procedure, a further improvement of the convergence properties has been observed in numerical experiments.

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