8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Bifurcation Analysis using Nonlinear Dynamics for the Multi-Folding Structures

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Key Words: *Bifurcation Analysis, Postbuckling, Nonlinear Dynamics, Multi Folding, Snap-through*

ABSTRACT

This paper reviews the theoretical basis for the dynamic numerical analysis to examine the elastic stability of a folding multi-layered truss[1]. The analysis allows for geometrical non-linearity and contact between nodes and is based upon bifurcation theory[2]. Comparisons are made between experimental folding patterns and the patterns obtained from the numerical method in which bifurcations are demonstrated as elastic unstable snap-through behaviour(ex.[3]). Several folding patterns are identified during the elastic instability where the folding behaviour of the truss is a function of the initial geometry and velocity of the dynamic loading. The authors suggest that the understanding of the behaviour will be very useful for the development of light weight structures subject to bifurcation.

Theoretical approach for multi-folding truss

We consider the folding mechanisms for the (pantographic) truss structure subject to a vertical impact load at the top node of the system[4]. The system is a pin-jointed elastic truss and all nodes of the system displace vertically only. No allowance is made for friction or gravity for this geometrically nonlinear problem.

We assume a periodic height for each layer of $h_i = \gamma_i L$ where the width L of the truss is fixed. Therefore, an initial length for each bar in the geometry of the figure is expressed as

$$\ell_i = \sqrt{L^2 + h_i^2} = L\sqrt{1+\gamma_i^2}, \quad \text{for } i = 1, \cdots, n. \tag{1}$$

Using definition of the Green strain, the total potential energy, $\mathcal{V}$, of the half model shown in Figure 1, subject to loading $f/2$ is then given by

$$\mathcal{V} = \sum_{i=1}^{n} \frac{EA_i\ell_i}{2}(\varepsilon_i)^2 - \frac{f}{2}\bar{v}_1 L \tag{2}$$

$$= \sum_{i=1}^{n} \frac{EA_iL\sqrt{1+\gamma_i^2}}{2}\frac{1}{4}\left\{\frac{1+(\gamma_i-\bar{v}_i+\bar{v}_{i+1})^2}{1+\gamma_i^2} - 1\right\}^2 - \frac{f}{2}\bar{v}_1 L. \tag{3}$$

For the case when $\gamma_i = \gamma,\ (i = 1, \cdots, n)$ and $EA_i = EA,\ (i = 1, \cdots, n)$ the total potential energy can be written as

$$\mathcal{V} = \frac{\beta L}{8} \sum_{i=1}^{n} (\bar{v}_i - \bar{v}_{i+1})^2 \left((\bar{v}_i - \bar{v}_{i+1}) - 2\gamma \right)^2 - \frac{f}{2} \bar{v}_1 L \qquad (4)$$

where the stiffness parameter $\beta = EA/(1+\gamma^2)^{3/2}$ (and therefore is a function of γ). From Eq. (4), we can obtain the equilibrium equations based on the principal of minimum energy in the following way:

$$F_i(\cdots, v_i, \cdots) \equiv \frac{\partial \mathcal{V}}{\partial v_i} = \frac{\partial \mathcal{V}}{\partial \bar{v}_i} \frac{\partial \bar{v}_i}{\partial v_i} = 0\,, \quad \text{for } i = 1, \cdots, n. \qquad (5)$$

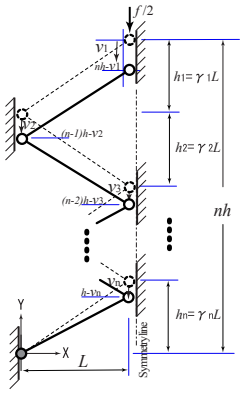

Figure 1: Folding model

Dynamic Analysis for Folding Truss

The dynamic analysis equation for the folding truss combines mass, damping and nonlinear stiffness $\{F_i(\boldsymbol{v})\}^{\mathrm{T}} = \boldsymbol{F}(\boldsymbol{v}) \in \mathbf{R}^N$ in the following equation:

$$M\ddot{\bar{\boldsymbol{v}}}(t) + C\dot{\bar{\boldsymbol{v}}}(t) + \boldsymbol{F}(\bar{\boldsymbol{v}}(t)) = 0$$

where, $M \in \mathbf{R}^{N \times N}$ is the mass matrix; $C \in \mathbf{R}^{N \times N}$ is the damping matrix; $\boldsymbol{F}(\cdot)$ is the nonlinear stiffness vector; $\{\ddot{\bar{v}}_i(t)\}^{\mathrm{T}} = \ddot{\bar{\boldsymbol{v}}}(t) \in \mathbf{R}^N$ is normalised acceleration; $\{\dot{\bar{v}}_i(t)\}^{\mathrm{T}} = \dot{\bar{\boldsymbol{v}}}(t) \in \mathbf{R}^N$ is the velocity; $\{\bar{v}_i(t)\}^{\mathrm{T}} = \bar{\boldsymbol{v}}(t) \in \mathbf{R}^N$ is the normalised displacement and N is the total number of degrees of freedom in the system. If the mass and damping in this system are given as independent uniform variables $m_i = m, c_i = c,\ (i = 1, \cdots, n)$, then we obtain the equation for the nodal variables $\ddot{\bar{v}}_1(t), \dot{\bar{v}}_1(t), \bar{v}_1(t)$ and this results for the displacement $\bar{v}_1(t)$ for node 1 as follows

$$m\ddot{\bar{v}}_1(t) + c\dot{\bar{v}}_1(t) + (\beta \mathcal{F}_1(\bar{v}_1) - f) = 0, \qquad (6)$$

dividing each term by m, we obtain the following equation

$$\ddot{\bar{v}}_1(t) + c'\dot{\bar{v}}_1(t) + \beta' \mathcal{F}_1(\bar{v}_1(t)) = f'(t) \qquad (7)$$

where $c' = c/m$, $\beta' = \beta/m$ and $f' = f/m$ (and includes both the primary path and the bifurcation loads), $f'(t)$ depends on time as the load control parameter. If the value of the damping parameter c' is small, the system response appears as vibration motion analogous to a molecular model.

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