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Construction of very high order residual distribution schemes for compressible flow problems.

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ABSTRACT

We are interested in the numerical approximation of steady hyperbolic problems

$$\text{div}\ \mathbf{f}(\mathbf{u}) = 0, \qquad u = g \text{ on the inflow boundary} \tag{1}$$

which are defined on an open set $\Omega \subset \mathbb{R}^d$, $d = 2, 3$ with weak Dirichlet boundary conditions defined on the inflow boundary[1]. In (1), the vector of unknown $\mathbf{u}$ belongs to $\mathbb{R}^p$, and the flux $\mathbf{f}$ is $\mathbf{f} = (f_1, \dots, f_d)$.

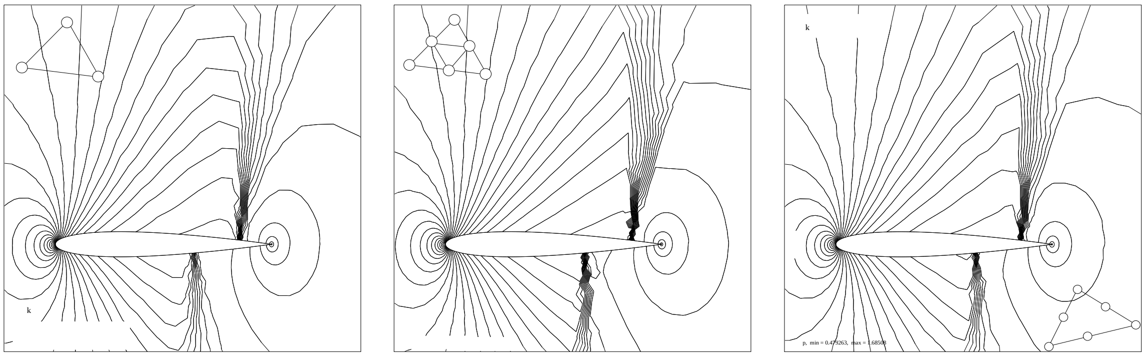
The target example we aim at is the system of the Euler equations with a perfec gas equation of state, but the method can be extended to other problems (see e.g. [1]). In the recent years, there have been many researches to produce really robust and high order schemes for equations of the type (1) and in particular for the Euler equations. In this paper we are concerned with the approximation of these equations on conformal unstructured meshes. We restrict our-self to the case of two dimensional triangular type meshes, even-though things can be made more general [2].

One may quote the class of ENO/WENO schemes and the class of Discontinuous Galerkin schemes. In the ENO/WENO case, (1) is approximated by a finite volume scheme where the entries of the flux are evaluated by a high order reconstruction polynomial. In our opinion, the main drawback of this approach is its algorithmic complexity and the non compact nature of the computational stencil : the average value of $\mathbf{u}$ in the cell $\mathcal{C}$ is updated by using its neighbors, and the neighbors of neighbors, and so on, depending on the expected accuracy. The non compactness of the stencil is also a serious drawback for the parallelisation of the code. In the case of DG schemes, the solution is approximated by a reconstruction function that is generally discontinuous across the interface of the elements K of the mesh. The solution of (1) is updated thanks to a local weak form of equation (1) combuned with a polynomial representation of the data in K, the flux through the edges of K are approximated by numerical flux functions. The formulation is very local and also very flexible but the number of degrees of freedom grows very quickly as the degree of the polynomial on K increases.

[1] $\vec{n}$ is the inward normal.

stays very local, as in the DG methods, but the number of degrees of freedom grows less quickly, even in 3D. The price to pay is to impose the continuity of the approximation $\mathbf{u}$ as in standard finite element methods. Indeed, the RD schemes can be seen as finite elements where the test functions may depend on the sought solution. This class of scheme is having a growing interest (see [3, 4, 5]) but has mainly been developed for second order accuracy only, see however [6] for different but related approach on structured meshes. In this paper, we are interested in showing how the methods we have developed in previous papers can be extended to very high accuracy, even in the case of the Euler equation, at a relatively moderate price.

During the conference, we will explain the implicit version of the scheme. Several computational examples will be presented with, hopefully, our first 3D results on an M6 wing.



second order | second order with fine mesh | Third order

Figure 1: Pressure on a Naca0012 airfoil, $M_\infty = 0.85$, 2° of incidence. Computed on a triangular unstructured mesh.

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