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# Effective Thermal Conductivity of Imperfect Carbon-Carbon Textile Composites Using the Mori-Tanaka Method

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## ABSTRACT

An efficient approach to the evaluation of effective thermal conductivity of carbon-carbon (C/C) plain weave textile composites using the Mori-Tanaka method is presented. The method proves its potential even if applied to real material systems with various types of imperfections including the non-uniform waviness of the fiber-tow paths. These are typically introduced in terms of histograms of the distribution of inclination angles [1]. Uncertainties linked to other geometrical parameters are mostly reflected in the derivation of an optimal shape of the equivalent ellipsoidal inclusion using a family of ideal periodic unit cells of Fig. 1(a).

Close inspection of actual micrographs of C/C composites reveals, apart from geometrical imperfections, a significant material porosity. Failure to account for the porous phase may result in severe overestimation of the predicted effective properties. This has been confirmed in [2] through detailed finite element simulations performed on a class of representative periodic unit cells shown in Fig. 1(b). Determination of such unit cells together with extensive finite element simulations is, however, rather tedious and time consuming. In some applications it therefore appears useful to substitute for these lengthy calculations by more efficient averaging techniques such as the Mori-Tanaka method, but properly accounting for possibly all sources of imperfections.

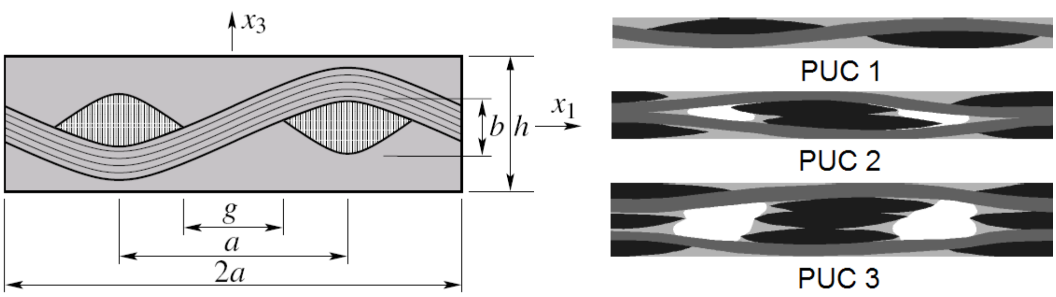


Figure 1: a) An idealized PUC, b) representative PUCs showing typical variation of porous phase

| Analysis | PUC | Representation of pores | | Thermal conductivity | |
|---|---|---|---|---|---|
| | | mutual ratio of size | $c_p$ | longitudinal | transverse |
| MT / FEM | PUC 1 | - | - | 9.38 / 9.46 | 1.63 / 2.27 |
| MT / FEM | PUC 2 | $4\times(150\times150\times30)$ | 0.108 | 8.24 / 9.03 | 1.12 / 1.47 |
| MT / FEM | PUC 3 | $8\times(150\times150\times30)$ | 0.141 | 7.90 / 7.29 | 1.00 / 1.53 |

Table 1: Effective thermal conductivities $\left[Wm^{-1}K^{-1}\right]$ (mesoscale)

The approach for the prediction of effective thermal conductivities of C/C composites presented hereafter follows the theoretical lines discussed in [1], but is extended to account for the porous phase.

Limiting our attention to a steady state heat conduction problem we write, in view of the Mori-Tanaka method assuming an ellipsoidal shape of the inhomogeneity, the local temperature gradient $\boldsymbol{H}$ as

$$\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{H}^f = \boldsymbol{H}_0 + \mathsf{S}\boldsymbol{H}^*, \tag{1}$$

where $\boldsymbol{H}_0$ is the average temperature gradient in the matrix and $\boldsymbol{H}^f$ represents the fluctuation part derived from and an equivalent inclusion problem [3]. Note that the second order tensor $\mathsf{S}$ and the vector $\boldsymbol{H}^*$ are analogous to the Eshelby tensor and transformation strain, respectively, for the elasticity problem. The Mori-Tanaka estimates are then provided by

$$\boldsymbol{K} = \mathsf{K}_0 + \left[\sum_{i=1}^{N} c_i \left(\mathsf{K}_i - \mathsf{K}_0\right)\langle\mathsf{P}_i\rangle\right]\left(c_0\mathsf{I} + \sum_{i=1}^{N} c_i\langle\mathsf{P}_i\rangle\right)^{-1}, \tag{2}$$

where index $0, 1, \ldots, i, \ldots, N$ refer to individual constituents with 0 reserved for the matrix phase and $c_r$ are the corresponding volume fractions. For a two phase model $P$ can be computed by means of the relation [3]

$$\boldsymbol{P} = \left[\mathsf{K}_0 + \left(\mathsf{K}_1 - \mathsf{K}_0\right)\mathsf{S}\right]^{-1}\left(\mathsf{K}_0 - \mathsf{K}_1\right), \tag{3}$$

The $\langle\cdot\rangle$ brackets represent the orientation averaging to grasp the fiber-tow paths on the mesoscale [1].

The results are presented in Tab. 1 assuming the porous phase being represented by an ellipsoid $\mathsf{S}$ with the dimensions obtained from a simple trial and error method. To arrive at a better agreement with finite element simulations, a search for an optimal shape of the equivalent ellipsoidal inclusion, much similar to that described in [1], is needed. Note rather different shape of voids for various PUCs, which was not taken into account in the MT predictions. This and the possibility of two-step homogenization, which may require to deal with an orthotropic matrix, are the topics of our current research.

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## REFERENCES

[1] J. Skoček, J. Zeman and M. Šejnoha. "Effective Properties of Carbon-Carbon Textile Composites: Application of the Mori-Tanaka Method". *Submitted to Mechanics of Materials*, 2007.

[2] B. Tomková, M. Šejnoha, J. Novák, and J. Zeman. "Evaluation of Effective Thermal Conductivity of Porous Textile Composites". *Submitted to Composites Part B: Engineering*, 2007.

[3] H. Hatta and M. Taya. "Equivalent inclusion method for steady state heat conduction in composites". *International Journal of Engineering Science*, Vol. **24**, 1159–1170,1986.