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A Coupled Discrete-Finite Element Method Modeling the Interaction of Granular Materials and Solid Structures

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ABSTRACT

A computational method for the simulation of granular materials and their interaction with solid structures is presented. The solids are modeled using continuum mechanical approaches and the finite element method while the granular materials are modeled by the discrete element method (DEM) [1]. Within the DEM particles of superellipsoidal [2] shape are applied, which are defined by five parameters through

$$F(\boldsymbol{x}) = \left(\left| \frac{x_1}{r_1} \right|^{\frac{2}{\epsilon_1}} + \left| \frac{x_2}{r_2} \right|^{\frac{2}{\epsilon_1}} \right)^{\frac{\epsilon_1}{\epsilon_2}} + \left| \frac{x_3}{r_3} \right|^{\frac{2}{\epsilon_2}} \leq 1, \tag{1}$$

where the r_i define the dimensions and ϵ_i control the roundness of the superellipsoid. This geometry description offers a variety of possible shapes, see Fig. 1 for some examples. The determination of forces between contacting particles is based on the HERTZIAN theory, whereas the tangential part is

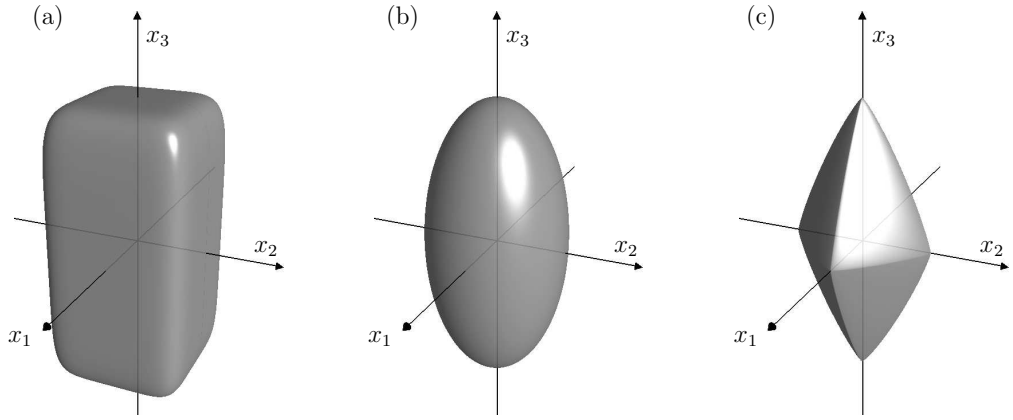

Figure 1: Superellipsoids with $r_1 = r_2 = r_3/2$ and $\epsilon_i = 0.3$ (a), $\epsilon_i = 1$ (b) and $\epsilon_i = 1.7$ (c).

modeled by COULOMB's friction law. Thus, the only constitutive parameters used in the model are the elastic constants of the particles and the inter-particle coefficient of friction.
To model the interaction with solid structures the DEM is coupled with the finite element method (FEM). Within a DEM analysis explicit time integration schemes are used to compute the particle trajectories hence an explicit transient analysis is applied on the FE side too. The interaction is accomplished via contact forces acting between the particles and the surface of the FE model. To reduce the computational effort this surface is approximated by a triangular mesh, whose vertices are the FE surface nodes. Contacts between particles and plane triangular surface patches are resolved by the HERTZIAN contact law again. For the FE analysis each force is then split into a static equivalent set of three parallel forces acting at the nodes. At edges or corners of the triangular mesh local smooth surface approximations are used so that the HERTZIAN contact law, which depends on the principal curvatures of the contacting surfaces, is still applicable.

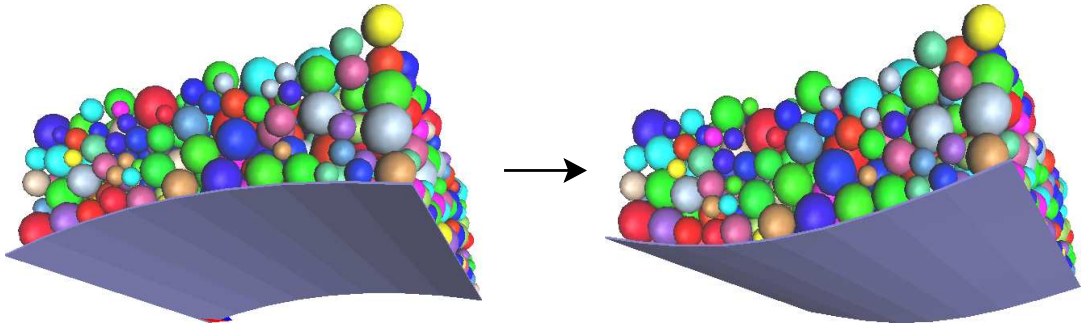

Figure 2: Example for coupled simulation: Snap through of curved shell due to particle load.

A problem of DEM simulations dealing with a huge number of particles is the high computational cost which is mainly due to the determination of all contact pairs within the particle assemblage. Here this effort is reduced by use of the concept of verlet lists [3] and an efficient contact algorithm, which explores the convexity of the particles and the common normal property at the contact points [4]. Furthermore, different time steps are enabled on the DEM and the FEM side to account for potentially large discrepancies between the critical time steps on both sides. Therefore, the time steps are chosen in a way that one time step is always an integer multiple of the other. At intermediate time steps interpolated configurations of the particles or the FE surface mesh, respectively, have to be used for the determination of particle-surface contact forces. Both time steps are controlled adaptively based on an energy balance criterion proposed in [5].

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