8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A POSTERIORI ERROR ESTIMATES FOR DISCONTINUOUS GALERKIN METHODS BASED ON WEIGHTED INTERIOR PENALTIES

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Key Words: *discontinuous Galerkin, a posteriori error analysis, singularly perturbed problems*

ABSTRACT

Most of the discontinuous Galerkin (DG) methods are usually defined by means of the so called numerical fluxes between neighboring mesh cells, see [1]. Nevertheless, for the interior penalty (IP) schemes for second order problems it is possible to correlate the expression of the numerical fluxes with a corresponding set of local interface conditions that are weakly enforced on each inter-element boundary. Such conditions are suitable to couple elliptic PDEs with smooth coefficients and it seems that a little attention is paid to the case of problems with discontinuous data or to the limit case of locally singularly perturbed problems, where the diffusivity vanishes in some parts of the computational domain.

Following [2], we look at the derivation of a DG scheme with a bias to domain decomposition techniques and we introduce an interior penalty DG method arising from a set of generalized interface conditions, proposed in [3] to couple both elliptic and hyperbolic problems and giving rise to the so called heterogeneous domain decomposition methods. Our purpose is to obtain a DG scheme that is *robust* for the approximation of problems that vary in character from one part of the domain to another, because of large variations of the coefficients.

In order to obtain such method, it is necessary to modify the numerical fluxes of a standard IP scheme, replacing the arithmetic mean with suitably weighted averages where the weights depend on the coefficients of the problem. This technique provides an advantage with respect to standard IP schemes because it enhances the capability of the method to regulate the degree of smoothness of the approximate solution with respect to the local variation of the coefficients of the problem. Furthermore, the weighted interior penalty scheme may find a practical application in the framework of homogenization theory, see [4]. Indeed, it can be exploited for the determination of the transport properties of porous media whose structure can not be described as a periodic distribution of obstacles.

In the present work, we focus our attention on the derivation of a posteriori local error indicators that are robust with respect to singularly perturbed problems. Following the framework proposed in [5] for the classical interior penalty scheme, we present suitable duality based error estimates for output functionals and for the L^2-norm. Then, we demonstrate how the introduction of weighted interior penalties allows

to incorporate into the scheme some a priori knowledge of the solution that improves the efficacy of the local error estimator and of the corresponding adapted mesh, leading to a more selective refinement than in the standard case.

Finally, we consider a residual based a posteriori error estimate in the energy norm, derived in the framework proposed in [6] for the Laplace equation. We compare this result with the ones obtained by duality, with particular attention to their behavior in the case of singularly perturbed problems.

All the theoretical results are illustrated and discussed by means of numerical experiments.

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