8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

OPTIMAL CONFIGURATION FOR DAMAGE IDENTIFICATION IN PIEZOELECTRICS

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Key Words: *Inverse Problems (IP), Finite Element Method (FEM), Probability of Detection (POD).*

1 Introduction

A numerical method to determine the location and extent of defects in piezoelectric plates is developed by combining the solution of an identification IP, using genetic algorithms to minimize a cost functional, and using an optimized FEM. The aim of this work is to find which excitation and measurement magnitudes provide the optimal configuration, in terms of high sensitivity to the defect and low sensitivity to noise and model uncertainties. An analytical estimation of the POD is developed and validated using Monte Carlo techniques. Finally, a number of excitation-measurement combinations are studied, estimating the POD.

2 Inverse Problem (IP)

The specimen consists of a PZT–4 ceramic plate ($L_x = L_z = 6$ [m]) with a defect, assumed to be represented by a circularly shaped cavity of radius r and centre (x_0,z_0). The defect characterization is performed in three steps: i) excitation of the specimen, ii) response (displacements or voltages) ψ_i measured at $N_i = 25$ points along the bottom of the plate , and iii) information is interpreted by the IP algorithm. The IP algorithm is developed in three steps: i) the cost functional, f, is defined as a quadratic difference between the experimental ψ_i^{EXP} and FEM–predicted ψ_i^{FEM} measurements: $f = \dfrac{1}{2N_i}\displaystyle\sum_{j=1}^{N_i}\left(\psi_i^{EXP} - \psi_i^{FEM}\right)^2$. ii) The output of the IP is a set of parameters need to characterize the defect. The parameters obtained as the estimated solution of the IP are grouped in a vector $\mathbf{p} = \{p_i\} = \{x_0, z_0, r\}$, whereas the parameters that represent the true characteristics of the defect are denoted by $\tilde{\mathbf{p}}$. iii) The IP of defect evaluation can be stated as a minimization problem, that may be constrained, as finding p_i such that, $\min_{p_i} f(\mathbf{p})$.

3 Probability of Detection (POD)

The POD gives an idea of the probability that the alteration of the measurement caused by the defect is larger than that caused by the noise. If the noise generator ξ_i is a random variable, the POD is a probability of the stochastic variable A^2 (square of the area of the defect), described by the cumulative probability density function F (see [1]),

$$POD = F\left(\frac{RMS^2\sigma^2\sum_{i=1}^{N_i}\xi_i^2}{S_A}\right) \tag{1}$$

Assuming this distribution, the squared sum of the noise ξ_i is known to follow a *Chi-square* distribution, since $\sum_{i=1}^{N_i}\xi_i^2 \longrightarrow \chi^2_{N_i}$. The parameter of the *Chi-square* distribution is the number of degrees of freedom N_i. If $N_i > 10$, the *Chi-square* distribution can be approximated by normal N distribution $\chi^2(N_i) \approx N(N_i - 2/3, \sqrt{2N_i})$ with mean $N_i - 2/3$ and standard deviation $\sqrt{2N_i}$. This approximation yields:

$$A^2 \longrightarrow N\left[\frac{RMS^2\sigma^2(N_i - 2/3)}{S_A}, \frac{RMS^2\sigma^2\sqrt{2N_i}}{S_A}\right] \tag{2}$$

Three Monte Carlo (MC) simulations are developed to validate and calculated the robustness of the POD's formulation obtained. Three samples of ξ_i are generated, with i) a normal random sample, ii) a sample containing the measurements that don't fulfill the *Kolmogorov-Smirnov* normality test with 5% significance level and iii) a sample outside the normality test with significance level 0.5%. The results are presented in Figure 1. Analytical and MC, using the sample i), curves match well, which allows to conclude the correctness of the analytical estimation of the POD, even for measurements not normally distributed.

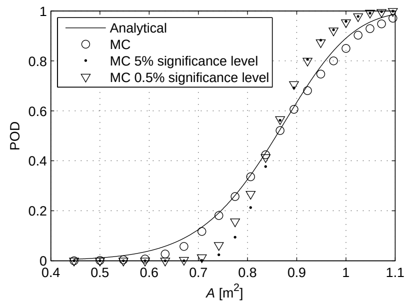

Figure 1: Comparison between analytical expression and Monte Carlo simulations of the POD

4 Results

In order to establish a quantitative comparison between excitation-measuring configurations, the minimal and median numerical values of the Relative Area detectable to POD=99.9% is developed for every configuration. Finally, this study concludes that the measurement of voltage combined with an excitation transversely to the polarization of the piezoelectric is the best configuration.

REFERENCES

[1] G. Rus, R. Palma and J.L. Pérez–Aparicio. “Optimal measurement setup for damage detection in piezoelectric plates”. *Int. J. for Num. Meth. in Eng.*, submitted, 2007.