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SOME ISSUES IN VISCOELASTIC HOMOGENEISATION WITH RESPECT TO THE COMPATIBILITY OF MICROSCOPIC SPECTRA

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ABSTRACT

Concrete is a heterogeneous material which exhibits a viscoelastic behaviour under a long term applied load. The heterogeneous nature of the material is retrieved at different scales of observation. At each scale, the representative element volume can be seen as elastic and porous inclusions embedded in a viscoelastic matrix. A robust descript of the macroscopic behaviour of the material suggests therefore to use upscaling techniques to built the homogenised behaviour. This breaks with the classical approaches in which macroscopic constitutive laws are constructed with several phenomenological parameters which are assumed, for instance, being able to describe the influence of the concrete mix on the creep behaviour. Indeed, homogenisation techniques incorporate naturally the microstructure associated with each concrete mix and then can make explicit its influence on the macroscopic behaviour.

Homogenisation of viscoelastic heterogeneous materials [2,3,4,5,7] can be investigated by using the correspondence principle [8] which permits to extend elastic homogenization schemes to the viscoelastic case. The correspondence principle consists in using the Laplace-Carson transform in order to make the viscoelastic problem equivalent to an elastic one in the transform space. The macroscopic viscoelastic properties (creep and relaxation functions) are therefore obtained in a straightforward manner but in the Laplace-Carson transform space. Thereafter, the difficulty consists in deriving theses properties in the time domain by inverting their Laplace-Carson transforms. Thus the inverse transformation makes it possible to determine the time evolution of the macroscopic viscoelastic functions.

When the viscoelastic behaviour, at the microscopic scale, of the phase which exhibits creep is described by discrete spectra (generalised Maxwell or generalised Kelvin analogical units) then the inverse transformation requires prior to determine the macroscopic retardation times which control the creep and relaxation kinetics [7].

These retardation times are given by the roots of polynomials whose degree depends on both of the number of the microscopic retardation times and the number of phases to be homogenised [7]. The issue here is then to carry out which conditions, if any, should be

satisfied by these roots in order to ensure that homogenisation upscaling preserves the features of viscolestic functions that follow from their spectral representation [8]. Indeed, the consequence of the spectral representation of any viscoelastic behaviour is that the time derivatives of the relaxation and creep functions are monotonic functions of time [6]. In the case of isotropic viscoelasticity, the volumetric and deviatoric creep functions should be monotonically increasing and concave functions.

In this contribution, it is shown, using Mori-Tanaka homogenisation schema [1,7,9], that the aforementioned roots should be real and negative in order to avoid hyperbolic-like macroscopic creep functions. Furthermore, using a two steps solution strategy, compatibility conditions of volumetric and deviatoric creep functions, at the microscopic scale, are addressed such that the macroscopic creep functions preserve their concavity. The compatibility conditions derive from the fact that macroscopic retardation times present as a family of sets of values, each set being bounded by two successive microscopic retardation times. Therefore, the roots to be determined coincide with the intersections of the sets related to the volumetric creep component with those of the deviatoric one. If the intersection is empty, then all the roots are complex and the concavity of the macroscopic creep function is lost. Note that, in the case of the simplifying but widely use assumptions [2,5,7], of a constant microscopic Poisson's ratio or bulk modulus of the viscoelastic phase, the compatibility of the microscopic spectra is not an issue since the viscoelastic behaviour in controlled by only one function, for instance the deviatoric one.

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