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MODULAR vs NON-MODULAR PRECONDITIONERS FOR FLUID-STRUCTURE SYSTEMS WITH LARGE ADDED-MASS EFFECT

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ABSTRACT

We address the numerical simulation of fluid-structure interaction (FSI) problems characterized by a strong added-mass effect. In coupled fluid-structure problems, the fluid acts over the structure as an extra mass, usually called added-mass, on the interface. The importance of the extra inertia term (due to the added-mass effect) in the structure equation increases with the ratio ρ_f/ρ_s, where ρ_f and ρ_s are the densities of fluid and structure, respectively. In the case of large added-mass effect, the setting up of effective FSI algorithms is difficult and classical iterative procedures are too slow or fail. We present different FSI algorithms and show how the added-mass effect affects their performances.

The Dirichlet-Neumann (DN) algorithm is a classical substructuring technique stemming from a domain decomposition viewpoint (see, e. g., [2]). It means that the fluid is solved by enforcing the velocity and the structure by enforcing the load at their common interface. Unluckily, as the added-mass effect becomes important, the usual DN scheme, even if improved by an acceleration strategy, is slow or may fail to converge. By means of Schur complements, the FSI problem can be reformulated as an interface displacement problem. It is well known that the DN scheme can be interpreted as preconditioned Richardson iterations over that interface problem. Thus, we call this method DN-Richardson. The DN preconditioner is modular, hence the subproblems can be solved using separate fluid and structure codes and the coupling only involves the transfer of information at the interface. To make the DN algorithm more effective, instead of performing Richardson iterations, we apply the GMRES algorithm to the preconditioned interface problem, yielding a DN-GMRES that is much faster and robust than DN-Richardson. However, its performance is still negatively affected by the added-mass.

A straightforward way to solve the FSI problem is to compute the solution of the FSI system (coming from the linearized problem discretized in time and space) without decoupling the fluid and the structure subproblems. In this approach we consider the FSI as a single problem, instead of two coupled

subproblems. Therefore, we employ matching grids and write the structure equation in term of structure velocity, so that the velocity is defined over the whole domain. Moreover, we use the same finite element space for fluid velocity, pressure and structure velocity (say, piecewise linear representation for all the scalar variables). This calls for a stabilized fluid formulation. Thanks to these choices, the coupling conditions (strong continuity of velocities and weak continuity of stresses) are easily imposed. We precondition this monolithic system in two steps. First, we apply a suitable scaling to the FSI system matrix, since the entries of the fluid and structure matrices have different magnitude. Then, the scaled system is preconditioned by an incomplete LU factorization (the ILUT preconditioner). Because of the non-symmetric nature of the system, we compute the solution by applying the GMRES or BiCGStab method to the preconditioned FSI system. We denote this approach by the name ILUT-GMRES or ILUT-BiCGStab. Our goal is to claim the efficiency of non-modular preconditioners for problems with large added-mass effect.

We also consider FSI algorithms based on inexact factorization techniques [4, 1]. The idea of these methods is to replace the FSI system matrix with inexact block-LU factors. This approximation involves a perturbation error, which can be reduced if the inexact factorization is carried out over the incremental system, but does not spoil the accuracy. The two step algorithm (L-step and U-step) can be easily rearranged in a semi-implicit procedure [3], which calculates the intermediate fluid velocity, solves the coupled pressure-structure system and corrects the fluid velocity. In particular, we take into consideration the so-called PIC methods, FSI counterpart of the pressure correction methods for pure fluid problems.

We choose to deal with the explicit treatment of the nonlinearities, both of the convective term and geometrical deformation, for all the methods mentioned above in order to focus on the fluid-structure coupling. The implicit treatment of the nonlinearities only requires to subiterate over the domain shape.

We apply all the methods to simulate the propagation of a pressure pulse in a straight pipe with deformable boundaries as the structure density varies. We consider both the fully 3D problem and its 2D approximation, obtained by intersecting the pipe with a plane. These simulations allow us to show that, while the number of subiterations for the DN-Richardson algorithm increases dramatically as the structure density approaches the fluid one, the number of subiterations for the DN-GMRES method increases only slightly. Simulations confirmed that the behavior of the ILUT-GMRES improves as the added-mass effect gets critical, making the method suitable for hemodynamics applications. Then, we compare the performances of the methods in terms of CPU-time on a realistic hemodynamics problem, the simulation of a pressure wave in the carotid bifurcation. The ILUT-GMRES method proved to be the fastest.

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