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An explicit space-time discontinuous Galerkin scheme with local time-stepping for unsteady flows

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ABSTRACT

During the ADIGMA project work on the space-time expansion discontinuous Galerkin (STE-DG) scheme with local time-stepping is performed. The scheme is arbitrary order accurate in space and time and can provide highly accurate solutions for a large class of important scientific problems.

We employ the usual discontinuous Galerkin approach where the solution is approximated by time dependant degrees of freedom multiplied with a set of space dependant basis functions and integration by parts. A Taylor expansion in space and time is used to define a space-time polynomial in each space-time element, as a predictor. The space derivatives are given by the approximate solution at the old time level, the time derivatives and the mixed space-time derivatives are computed from these space derivatives using the so-called Cauchy-Kovalevskaya procedure. The space-time volume integral is approximated by Gauss quadrature with values at the space-time Gaussian points obtained from the Taylor expansion. The flux in the surface integral is approximated by a numerical flux with arguments given by the Taylor expansions from the left and from the right hand side of the element interface, see [2] and [3].

Within the ADIGMA project the focus lies on the improvement of two main aspects:

Increase of efficiency in the temporal and spatial discretization. The scheme's stencil itself is very compact, because each cell requires only flux data from the von Neumann neighborhood. By means of the space-time prediction, we can give up the assumption that all grid cells run with the same time step and introduce local time-stepping. We do not have any longer a common time level. Each cell may evolve in time with its own local time step Δt_i^n which has to satisfy a local stability restriction, depending on the grid cell diameter as well as on the order of accuracy $p+1$, see [3]. The cells are only synchronized at certain pre-defined time levels for output.

The shock capturing property. For problems where shocks and other discontinuities are involved, our discontinuous Galerkin scheme like any other high order scheme requires a special strategy in order to avoid Gibbs-type oscillations. First a strategy for detecting these troubled zones or cells is needed. Several so-called troubled cell indicators are available in order to find and flag those cells. For our STE-DG scheme the most useful indicators are the so-called jump indicator according to Qiu and Shu [5] and the indicator according to Persson and Peraire [4]. Then, one aims at preventing the numerical scheme from producing spurious oscillations in the vicinity of these regions. Hereby, our goal is to minimize the amount of data from the neighboring elements next to the troubled zone for two reasons:

High order discontinuous Galerkin schemes typically make use of large cells. Thus, if too much data from large neighbor elements is required for a smooth reconstruction of the troubled zone, the precision of the solution cannot be maintained. And secondly, when using the local time-stepping framework, obtaining neighbor information can cause problems or may lead to inefficiency. We have adopted the artificial viscosity approach as described in [4] to our explicit unsteady scheme STE-DG [1]. Where we keep relatively large grid cells and try to resolve the shock within a few of them forming a narrow viscous profile by locally adding some sort of artificial viscosity. The amount of viscosity, however, can be estimated from the grid properties, the value given by the indicator and an additional parameter.

We show high order calculations of two-dimensional test cases from the ADIGMA test case suite. Among these are the subsonic and transonic flow around a NACA0012 profile and the two-dimensional double Mach reflection test case of Woodward and Colella [6], in order to demonstrate the shock capturing property.

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