8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

MODELLING AND VARIATIONAL ANALYSIS OF AN ELECTRO-VISCOELASTIC CONTACT PROBLEM

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Key Words: *Electro-Viscoelastic Material, Frictionless Contact, Normal Compliance, Electrical Conductivity Coefficient, Variational Inequality, Monotone Operator, Fixed Point, Weak Solution.*

ABSTRACT

The piezoelectric effect is characterized by the coupling between the mechanical and electrical properties of the materials. This coupling leads to the appearance of electric potential when mechanical stress is present, and conversely, mechanical stress is generated when electric potential is applied. The first effect is used in mechanical sensors, and the reverse effect is used in actuators. A deformable material which exhibits such a behavior is called a piezoelectric material. Piezoelectric materials for which the mechanical properties are viscoelastic are also called electro-viscoelastic materials. Currently there is a considerable interest in frictional or frictionless contact problems involving piezoelectric materials. see for instance [1, 2, 3] and the references therein.

In this paper we present a mathematical model which describes the contact between an electro-viscoelastic body and an electrically conductive foundation. Our interest is to describe a physical process in which both contact, electrical conductivity, inertial and piezoelectric effects are involved, and to show that the resulting model leads to a well-posed mathematical problem. Taking into account the conductivity of the foundation leads to new and nonstandard boundary conditions on the contact surface, which involve a coupling between the mechanical and the electrical unknowns, and represents one of the traits of novelty of this work.

The physical setting is as follows. An electro-viscoelastic body occupies the domain $\Omega \subset \mathbb{R}^d$ $(d = 2, 3)$ with a smooth boundary $\partial\Omega = \Gamma$. We assume a partition of Γ into three open disjoint parts Γ_1, Γ_2 and Γ_3, on the one hand, and a partition of $\Gamma_1 \cup \Gamma_2$ into two open parts Γ_a and Γ_b, on the other hand. The body is acted upon by body forces and volume free electric charges. Moreover, it is clamped on Γ_1 and surface tractions act on Γ_2. We also assume that the electrical potential vanishes on Γ_a and a surface electrical charge is prescribed on Γ_b. In the reference configuration the body may come in contact over Γ_3 with an obstacle, the so called foundation. The contact is frictionless and is modeled with the normal compliance condition, in a form with a gap function; also, the foundation is assumed to be electrically conductive and therefore there may be electrical charges on the part of the body which is in contact

with the foundation and which vanish when contact is lost. The process is assumed to ne mechanically dynamic and electrically static.

We denote by $[0,T]$ the time interval of interest and we consider the spaces

$$V = \{\, \boldsymbol{v} \in H^1(\Omega)^d \ :\ \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma_1 \,\}, \quad W = \{\, \psi \in H^1(\Omega) \ :\ \psi = 0 \text{ on } \Gamma_a \,\}$$
$$\mathcal{H} = \{\, \boldsymbol{\tau} = (\tau_{ij}) \ :\ \tau_{ij} = \tau_{ji} \in L^2(\Omega) \,\} = L^2(\Omega)^{d\times d}_{sym}.$$

Below, $\boldsymbol{\varepsilon}$ and ∇ represent the deformation and gradient operators, respectively, and the dots above denote derivatives with respect to time; $(\cdot,\cdot)_X$ is the inner product of the space X, V' is the dual of V and $\langle\cdot,\cdot\rangle_{V'\times V}$ represents the duality pairing between V' and V.

With this notation, the variational formulation of the electro-viscoelastic problem can be formulated as follows.

Problem $\mathcal{P}$**.** *Find a displacement field* $\boldsymbol{u} : [0,T] \to V$ *and an electric potential* $\varphi : [0,T] \to W$ *such that*

$$\langle \ddot{\boldsymbol{u}}(t), \boldsymbol{v}\rangle_{V'\times V} + (\mathcal{A}\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}(t)), \boldsymbol{\varepsilon}(\boldsymbol{v}))_{\mathcal{H}} + (\mathcal{B}\boldsymbol{\varepsilon}(\boldsymbol{u}(t)), \boldsymbol{\varepsilon}(\boldsymbol{v}))_{\mathcal{H}} + (\mathcal{E}^*\nabla\varphi(t), \boldsymbol{\varepsilon}(\boldsymbol{v}))_{\mathcal{H}} \\ + j(\boldsymbol{u}(t), \boldsymbol{v}) = \langle \boldsymbol{f}(t), \boldsymbol{v}\rangle_{V'\times V} \quad \forall \boldsymbol{v} \in V,\ \text{a.e. } t \in (0,T), \tag{1}$$

$$(\boldsymbol{\beta}\nabla\varphi(t), \nabla\psi)_{L^2(\Omega)^d} - (\mathcal{E}\boldsymbol{\varepsilon}(\boldsymbol{u}(t)), \nabla\psi)_{L^2(\Omega)^d} + h(k, \boldsymbol{u}(t), \varphi(t), \psi) \\ = (q(t), \psi)_W \quad \forall \psi \in W,\ t \in [0,T], \tag{2}$$

$$\boldsymbol{u}(0) = \boldsymbol{u}_0, \quad \dot{\boldsymbol{u}}(0) = \boldsymbol{v}_0. \tag{3}$$

Here $\mathcal{A}$, $\mathcal{B}$, $\mathcal{E}$ and $\boldsymbol{\beta}$ are operators which describe the viscous, the elastic, the piezoelectric and the electric permittivity properties of the material, respectively; $\mathcal{E}^*$ is the transposite of the operator $\mathcal{E}$ and j represents the contact functional; the functions $\boldsymbol{f} : [0,T] \to V'$ and $q : [0,T] \to W$ in (1)–(2) are related to the force and electrical charge densities, respectively; the functional h in (2) describes the electrical condition on the contact surface Γ_3 in which k denotes the electrical conductivity coefficient. Finally, the data $\boldsymbol{u}_0$ and $\boldsymbol{v}_0$ in (3) represent the initial displacement and velocity field, respectively.

We prove the existence of a unique weak solution to problem $\mathcal{P}$. The proof is based on arguments of evolution equations with maximal monotone operators and fixed point. We also investigate the behavior of the solution with respect to the electrical conductivity coefficient, derive various estimates and prove a convergence result. We then consider a fully discrete approximation of the problem, based on the augmented Lagrangian approach and the finite element method. We implement this scheme in a numerical code and present numerical simulations which confirm our theoretical convergence result.

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