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MEMS MODELLING USING NON-CONFORMING ELEMENTS

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ABSTRACT

In this work different modelling techniques are investigated to simulate the dynamic behaviour of slender structures on which electrostatic forces are acting. In particular, non-conforming elements are tested to model micro-mechanical devices (or MEMS) having a very large aspect ratio. The theory underlying the non-conforming approach for finite elements is outlined in reference [1]. The principle of this formulation consists in enhancing the linear displacement field by quadratic shape functions associated to internal degrees of freedom. For a three-dimensional element, the displacement field $\mathbf{u}$ of components u_i $(i = 1, 2, 3)$ is discretised as :

$$u_i = \mathbf{M}\mathbf{U}_i + \mathbf{N}\mathbf{A}_i \quad \text{where} \quad \left\{ \begin{array}{l} \mathbf{N} = [N_1 N_2 N_3] \\ \mathbf{M} = [M_1 M_2 M_3 M_4] \end{array} \right. \text{and} \left\{ \begin{array}{l} N_1 = (1-\xi^2) \\ N_2 = (1-\eta^2) \\ N_3 = (1-\chi^2) \end{array} \right. \qquad (1)$$

where M_i are the linear shape functions (*e.g.* [2]), N_i the enhanced quadratic shape functions and ξ, η, χ $\in [-1, 1]$ the normalized coordinates in the reference space of the element. $\mathbf{U}_i$ is the nodal displacement array for direction i, and $\mathbf{A}_i$ are the internal unknowns. The equilibrium between internal and external force depends on both unknowns $\mathbf{U}_i$ and $\mathbf{A}_i$, but a Guyan's reduction may be applied on the internal degrees of freedom for each element. Eliminating the internal degrees of freedom yields the stiffness matrix related to the sole nodal degrees of freedom.

As a consequence the element compatibility is not exactly satisfied, but such elements can efficiently model beam- or shell-like structures with a small number of degrees of freedom. The advantage of non-conforming elements compared to shell or beam elements is that they are volume elements and can therefore easily be combined with other volume finite elements. For micro-mechanical systems the structure must be coupled to the electrostatic domain with the so-called electro-mechanical elements that solve for the electrostatic potential and generate the electrostatic forces. This cannot be easily

achieved for the extremities of beam or shell elements. This research shows that constructing coupled electro-mechanical models for high aspect ratio systems is then greatly simplified when non-conforming finite elements are used. The theory is presented for small deformations and also for large displacements where geometric non-linearities must be accounted for.

The efficiency of the non-conforming approach combined with specific electro-mechanical elements (developed by Rochus *et al.* [3]) is highlighted in the analysis of two simple MEMS for which the pull-in voltage is computed. Non-conforming elements appeared to be the method of choice combining accurate modelling of slender structures and proper assembly with electro-mechanical elements. Hence, as a conclusion we recommend using non-conforming elements for most of practical problems where the electrostatic field can be modelled with linear elements. If quadratic electrostatic elements are used, using quadratic structural elements seems then more appropriate in order to ensure interface compatibility. It was found that non-conforming elements outperform quadratic elements since they allow reaching the same level of accuracy for significantly less degrees of freedom. In the benchmark presented in figure 1, it was shown that the modelling technique for electro-mechanical systems allows computing pull-in curves that are very similar to the measured results.

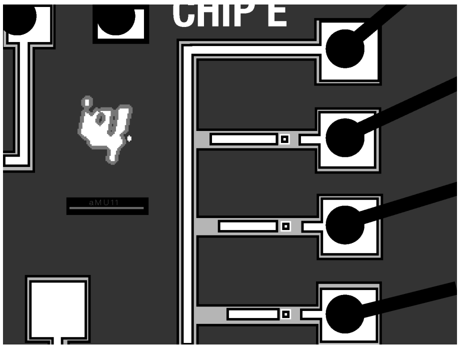

Figure 1: Cantilever Micro-beams.

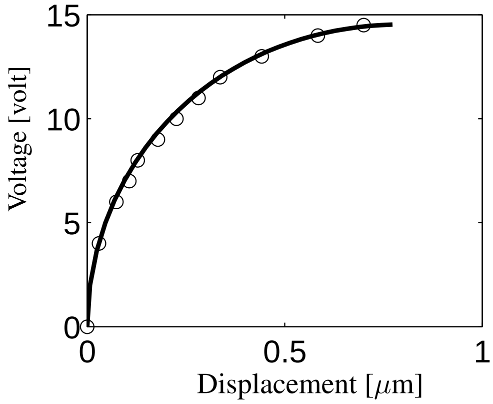

Figure 2: Pull-in curve for the cantilever beam. The plain line is obtained numerically and circles represent the experimental data.

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