8th World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

EFFICIENT SIMULATION OF CONTACT PROBLEMS: MULTI-BODY CASE AND LARGE DEFORMATIONS

*** Thomas Dickopf[1], Rolf Krause[2], and Christina Mohr[3]**

[1,2,3] Institute for Numerical Simulation, University of Bonn, Wegelerstraße 6, 53115 Bonn, Germany
[1] *dickopf@ins.uni-bonn.de* [2] *krause@ins.uni-bonn.de* [3] *mohr@iam.uni-bonn.de*

Key Words: *contact problems, multigrid methods, constrained minimization, large deformations, level-set approach, non-matching meshes, domain decomposition*

ABSTRACT

Considering the mechanical behavior of elastic bodies, one has to deal with several non-linear effects. We present a methodical approach for a flexible numerical simulation in mechanics, which suits various problem classes. Here, the main focus is on the efficient discretization and numerical solution of single- and multi-body contact problems in linear and non-linear elasticity on complex geometries in 3D.

In the mathematical description of a single-body contact problem, the originally global non-linearity, i. e. non-penetration of the body and the rigid obstacle, is reformulated as a set of pointwise inequality constraints. More precisely, if we assume linear elastic material behavior, the displacement field $\boldsymbol{u} : \Omega \to \mathbb{R}^3$ is given as the solution of the free boundary value problem

$$\begin{array}{rcll} -\mathbf{div}\,(\boldsymbol{\sigma}(\boldsymbol{u})) & = & \boldsymbol{f} & \text{in}\,\Omega, \\ \boldsymbol{u} & = & \mathbf{0} & \text{on}\,\Gamma_D, \\ \boldsymbol{\sigma}(\boldsymbol{u})\boldsymbol{n} & = & \boldsymbol{p} & \text{on}\,\Gamma_N, \end{array} \tag{1}$$

with the contact conditions

$$\sigma_n(\boldsymbol{u}) \leq 0, \quad \boldsymbol{\sigma}_t(\boldsymbol{u}) = \mathbf{0}, \quad \boldsymbol{u}\cdot\boldsymbol{n} \leq d_n, \quad (\boldsymbol{u}\cdot\boldsymbol{n} - d_n)\sigma_n(\boldsymbol{u}) = 0 \quad \text{on}\,\Gamma_C. \tag{2}$$

Here, we decompose the boundary $\partial\Omega = \overline{\Gamma}_D \cup \overline{\Gamma}_N \cup \overline{\Gamma}_C$, denote the outer normal by $\boldsymbol{n}$, and measure the initial distance in direction $\boldsymbol{n}$ between Γ_C and the obstacle by the smooth gap function d_n. Furthermore, $\boldsymbol{f}$ and $\boldsymbol{p}$ are given force densities, $\boldsymbol{\sigma}$ is the linear stress tensor, and we have $\boldsymbol{\sigma n} = \sigma_n \boldsymbol{n} + \boldsymbol{\sigma}_t$.

It is well-known that (1), (2) can equivalently be written as a constrained minimization problem of a quadratic energy functional $\mathcal{J}$ over the convex set $\mathcal{K}$ of admissible displacements and is uniquely solvable. Then, the problem (1), (2) can be discretized using standard finite element techniques. The emerging system is solved by an adaptive monotone multigrid method of optimal complexity, see [3], which does neither use regularization nor an outer iteration but, instead, deals with the constraints by a non-linear block Gauß-Seidel smoother on the finest grid. We employ parallel computing.

In case of large deformations $\boldsymbol{\varphi} = \mathbf{id} + \boldsymbol{u}$ and non-linear material behavior, the energy functional $\mathcal{J}$ becomes non-convex. Besides, a linearization of the non-penetration condition as in (2) cannot effectively

be used. Hence, we choose a level-set approach to compute a signed distance function $d : \mathbb{R}^3 \to \mathbb{R}$, which measures the distance to the rigid obstacle, and enforce the exact geometric contact condition by

$$g(\boldsymbol{\varphi}) := \max_{\overline{\Omega}} (d \circ \boldsymbol{\varphi}) \leq 0.$$

Note that, here, the function g representing the constraints is neither affine nor smooth in general. Then, the resulting non-convex constrained minimization problem can be solved by a non-smooth SQP method applying the presented multigrid method to the constrained quadratic subproblems with the generalized derivative ∂g as obstacle direction.

In the multi-body case, i. e. $\Omega = \Omega^m \cup \Omega^s$ and, hence, $\boldsymbol{u} = (\boldsymbol{u}^m, \boldsymbol{u}^s)$, the non-linearity due to contact has a completely different appearance. The contact conditions for the transmission of forces between two deformable bodies read as

$$\sigma_n(\boldsymbol{u}^m \circ \boldsymbol{\Phi}) = \sigma_n(\boldsymbol{u}^s) \leq 0, \quad [u] \leq d^{\Phi}, \quad ([u] - d^{\Phi})\sigma_n(\boldsymbol{u}^s) = 0 \quad \text{on } \Gamma_C^s,$$

where $\boldsymbol{\Phi}$ is a bijective contact mapping and $[u] := (\boldsymbol{u}^s - \boldsymbol{u}^m \circ \boldsymbol{\Phi}) \cdot \boldsymbol{n}^{\Phi}$ is the jump in $\boldsymbol{n}^{\Phi}$-direction. The normal field $\boldsymbol{n}^{\Phi}$ and the inital gap d^{Φ} are defined by $\boldsymbol{\Phi}$, which is a priori unknown and usually approximated by suitable normal projections in case of a linear material.

The discretization of the coupling constraints is done in a weak sense by a modified mortar finite element method, see [1], proposing the use of an L^2-projection between non-matching meshes to allow for optimal error estimates. Here, we develop an efficient method to compute the emerging discrete transfer operator for complex geometries. We compute triangulated intersections of opposite element faces in a locally adjusted projection plane but carry out the necessary numerical quadrature on possibly warped contact boundaries directly, see [2, 4]. Then, the transfer operator is used to perform a local basis transformation so that, again, the non-linear multigrid solver [3] can be applied. To simulate “glued” interfaces, we can also prescribe equality constraints.

Our approach has to be recognized in the much more general context of (non-conforming) domain decomposition methods. A stable and efficient information transfer, not only between non-matching meshes in 2D and 3D but also between different discretizations and even different models, is crucial for the proof of approximation error estimates as well as of convergence properties of iterative solvers.

The simulation of multi-body contact with large deformations requires the incorporation of the discrete coupling operator into the outer iteration. Since the local coupling is only reasonable for small deformations, this could imply additional box constraints in each SQP step and, thus, affect convergence. In addition, the signed distance function has to be recomputed during the iteration.

Finally, we illustrate that our methods are well suited for the simulation of other contact phenomena such as Coulomb friction and thermoelasticity. Moreover, we consider dynamic problems with biphasic materials and a system of Allen-Cahn type equations coupled with elasticity.

REFERENCES

[1] C. Bernardi, Y. Maday, A. T. Patera. A new nonconforming approach to domain decomposition: the mortar element method. In *Nonlinear partial differential equations and their applications,* H. Brezis, J. L. Lions (eds). Pitman Research Notes in Mathematics 299. Harlow: Longman Scientific & Technical: New York, 1994; 13–51.

[2] T. Dickopf, R. Krause. Efficient simulation of multi-body contact problems on complex geometries: A flexible decomposition approach using constrained minimization. INS Preprint 0711, University of Bonn, 2007.

[3] R. Kornhuber, R. Krause. Adaptive multigrid methods for Signorini's problem in linear elasticity. *Computing and Visualization in Science* 2001; **4**(1):9–20.

[4] M. A. Puso. A 3D mortar method for solid mechanics. *International Journal for Numerical Methods in Engineering* 2004; **59**:315–336.