

NONLINEAR INTEGRAL EQUATIONS FOR A 3D INVERSE ACOUSTIC SCATTERING PROBLEM

* O. Ivanyshyn¹ and M. Pieper²

¹ Institute for Numerical and Applied Mathematics, Georg-August University of Göttingen, Lotzestr. 16–18, 37083 Göttingen, Germany
ivanyshy@math.uni-goettingen.de

² Fraunhofer Institute for Industrial Mathematics, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany
pieper@itwm.fraunhofer.de

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ABSTRACT

In many practical applications, such as radar, sonar, geophysical exploration or medical imaging, one tries to detect an obstacle by using scattering phenomena. We present an iterative method for solving the inverse scattering problem, where the shape and the location of the 3 dimensional sound-soft obstacle are unknown and should be reconstructed from the knowledge of incident plane waves and the corresponding far field patterns. The approach originates from a method suggested by Kress and Rundell [2] for an inverse boundary value problem for the Laplace equation and was successfully transferred to 2d acoustic scattering problems in 2006 (see [1]). The idea of the method is the following. We reformulate the inverse problem as a system of two nonlinear boundary integral equations for the unknown boundary curve and the normal derivative of the total field on the boundary. These integral equations can be solved by linearization, i.e., by regularized Newton iterations. The Fréchet derivatives with respect to the boundary curve are represented again as integral operators (see Potthast 1994). The numerical treatment of the integral operators in 3d is more complicated, therefore 3d problem is more challenging. To discretize operators with singularities in the kernels we apply Wienert's method (see [3,4]) and we use the Gauss rectangular rule for the operators with smooth kernels. Then Galerkin method is employed to solve the system of two equations. Since the linearized equations inherit ill-posedness of the inverse problem a regularization technique has to be incorporated, e.g. Tikhonov regularization.

REFERENCES

- [1] O. Ivanyshyn and R. Kress. "Nonlinear integral equations in inverse obstacle scattering". In: *Mathematical Methods in Scattering Theory and Biomedical Engineering (Fotiadis, Mas-salas, eds)*. World Scientific, Singapore, 39–50, 2006.
- [2] R. Kress and W. Rundell. "Nonlinear integral equations and the iterative solution for an inverse boundary value problem". *Inverse Problems*, Vol. **21**, 1207–1223, 2005.
- [3] M. Pieper. "Spektralrandintegralmethoden zur Maxwell-Gleichung". PhD-Thesis, Göttingen, 2007.
- [4] L. Wienert. "Die numerische Approximation von Randintegraloperatoren für die Helmholtzgleichung im \mathbb{R}^3 ". PhD-Thesis, Göttingen, 1990.