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AN EXTENDED STOCHASTIC FINITE ELEMENT METHOD TO DEAL WITH GEOMETRICAL UNCERTAINTIES IN STRUCTURAL ANALYSIS

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ABSTRACT

In a structural analysis, the incorporation of uncertainties, related to material properties, loadings or geometry, seems today essential if one seeks to obtain “reliable” numerical predictions, usable in a design process or a decision-making. This necessity have led to a rapid development of many ad hoc numerical techniques, such as Galerkin stochastic finite element methods [1,2]. These methods provide high quality predictions which are explicit in terms of the random variables describing the uncertainties. The incorporation of uncertainties on material properties or loadings is quite well mastered within the framework of these techniques. However, there is still no available efficient strategy to deal with uncertainties on the geometry although it could have a great interest in various applications.

A natural way to solve a stochastic problem involving geometrical uncertainties consists in coupling a simple deterministic finite element calculation code with a classical stochastic method such as Monte-Carlo simulations, response surface method... However, these techniques require numerous deterministic finite element computations, each of which requiring the construction of a new conforming mesh, which leads to prohibitive computational costs. Moreover, this kind of strategy with remeshings doesn’t allow the obtention of an explicit description of the solution.

Recently, a Galerkin stochastic finite element method has been proposed for solving partial differential equations on random domains [3]. This method, called eXtended Stochastic Finite Element method (X-SFEM), is an extension to the stochastic framework of the X-FEM method [4]. It does not require any remeshing and allow an easy handling of complex geometries.

The first point of the method consists in using an implicit description of the geometry with the level-set technique [5]. Geometrical uncertainties are then characterized by random level-set functions, which are discretized on a fixed mesh of a fictitious deterministic domain. The second point consists in using a fully Galerkin approximation technique at both deterministic level (using finite element approximation) and stochastic level (using generalized polynomial chaos expansions).

In this presentation, we recall the basis of the X-SFEM method, initially developed for the case of random shapes, and briefly address computational aspects (related to the construction and resolution of

the discretized problem). Then, the method is extended to the case of material interfaces. In particular, we address the question of enrichment of the approximation space by the partition of unity method [6]. This enrichment, proposed in [7] for the deterministic X-FEM method, introduce discontinuities in the derivatives of the solution at the random material interface. It allows recovering classical convergence rates of finite element approximations. Numerical examples will illustrate the quality of the obtained solution and the efficiency of the method when compared to classical resolution techniques.

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