**8th. World Congress on Computational Mechanics (WCCM8)**
**5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)**
**June 30 –July 5, 2008**
**Venice, Italy**

# THIRD–ORDER METHOD FOR COMPRESSIBLE FLOWS ON UNSTRUCTURED MESHES

**J. Rokicki[1], R. Wieteska[2]**

[1] Institute of Aeronautics
and Applied Mechanics
Nowowiejska 24,
00-665 Warsaw, Poland
jack@meil.pw.edu.pl

[2] Institute of Aeronautics
and Applied Mechanics
Nowowiejska 24,
00-665 Warsaw, Poland
wietek@meil.pw.edu.pl

**Key Words:** *High–order methods, unstructured meshes, reconstruction, transonic flows.*

## ABSTRACT

Finite Volume method provides straightforward conservative discretisation for Euler equations. In its basic form the control volumes coincide with grid cells, while the unknowns are located at the cell centers. In order to calculate fluxes at the cell walls, the solution is reconstructed within each cell using the information from its neighbourhood. The order of spatial discretisation depends on two factors: (i) the order of reconstruction within the cell, (ii) the order of integration formula used at the cell walls to calculate fluxes.

The main objective of this work is presentation of the new reconstruction procedure (third order of accuracy in space) together with the procedure for integration of numerical fluxes applied to simulations of compressible 3D Euler flows using unstructured meshes.

On each grid cell each solution component is expressed as:

$$\phi\left(\mathbf{r}\right) = \phi_0 + \left(\mathbf{r}_p - \mathbf{r}_0\right)^T \cdot \nabla\phi\Big|_0 + \frac{1}{2}\ \left(\mathbf{r}_p - \mathbf{r}_0\right)^T \cdot \nabla^2\phi\Big|_0 \cdot \left(\mathbf{r}_p - \mathbf{r}_0\right) \tag{1}$$

The third–order gradient and hessian has to be calculated respectively with second and first order accuracy. These values are obtained as linear combination of function increments between cells in the computational stencil (see fig. 1)

$$\mathbf{L}^{(n)}\phi = \sum_{p=1}^{m} \mathbf{L}_p^n \cdot \omega_p \cdot \left(\phi_p - \phi_0\right) \tag{2}$$

where $\mathbf{L}^{(1,2)}$ stands for the gradient and hessian respectively. The unknown coefficients (vectors for $n = 1$, matrices for $n = 2$) are obtained by expanding increments into Taylor series and substituting them into (1). The resulting linear equation system is underdetermined so it's solved in the least square sense by a sequence of Householder transformations.

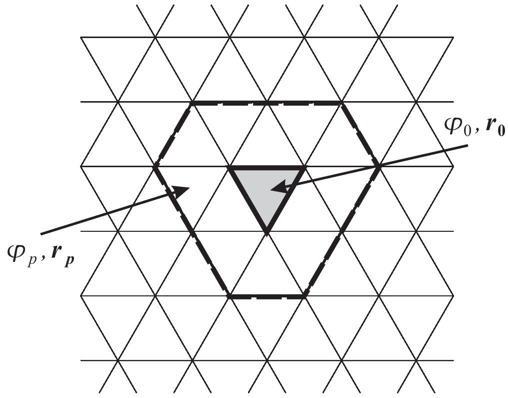


Figure 1: Typical stencil used in the reconstruction process.

In the case of transonic flows in which the shock–waves appear, the high–order method was coupled with WENO (Weighted Essentially Non Oscillatory) nonlinear mechanism which suppresses unphysical oscillations appearing during the iteration process.

The idea of the WENO schemes consist in continuous switching between different stencils in order to evaluate correct reconstruction of the solution inside each mesh cell (more details in [1],[2]).

$$\phi = \sum_i \omega_i \cdot \phi_i \tag{3}$$

In the present approach apart from the central stencil 4 biased stencils were used (3 in the 2D case). It was shown that this approach is superior to those based on switching to lower–order formulas. The performance of the proposed method was investigated for three types of examples:

- 3D channel flows (subsonic and transonic),
- ONERA M6 wing,
- wing–body system.

In the simplest case of the channel flow it was possible to prove by means of grid convergence study that the method indeed is close to 3$^{\text{rd}}$ order accuracy. For the remaining cases it was shown that the third–order method generates significantly lower amount of spurious entropy than its second–order counterpart. In addition it was positively verified that the method remains insensitive to large deformation of grid cells. In the relevant experiment a selected region of the mesh was deformed in anisotropic manner with the deformation ratio reaching up to 1/100 (in one direction only). The numerical solution on the deformed grid was virtually unchanged with respect to the reference grid.

It was also shown that the numerical cost of the third–order method is twice as high as the standard 2$^{\text{nd}}$ order WENO method. The number of iteration of the flow solver was in both cases almost the same.

## REFERENCES

[1] J. Rokicki and R. Wieteska. “Third–Order WENO Method on 3D Unstructured Tetrahedral Meshes”, *Journal of Transdisciplinary System Science,* Vol. **11**, 74–85, 2006.

[2] Ch. Hu and Ch-W. Shu. “High Order ENO and WENO Schemes for Computational Fluid Dynamics”, *Journal of Computational Physics*, Vol. **150**, 97–127, 1999.