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Implicit Harmonic Balance Solver for Forced Motion Transonic Flow

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ABSTRACT

Flight simulation requires the creation of an aerodynamic model that relates the aircraft state with the forces and moments. The model has historically been built on wind tunnel testing. An important part of the aerodynamic database is for the damping derivatives, that is forces and moment contributions arising from angular rates. These are derived from periodic forced motions. With advances in computational fluid dynamics there is potential for developing flight dynamics aerodynamic models from simulation. This is particularly attractive in the transonic regime where wind tunnel testing can be particularly expensive.

For the calculation of dynamic derivatives the requirement of multiple unsteady calculations is potentially costly in terms of CPU time. However, it is possible to exploit the periodicity of the problem to reduce the cost to something approaching that of steady state calculations. This approach was adopted in [1], using the Harmonic Balance method originally introduced for turbomachinery applications [2]. Previous work has used an explicit solution of the harmonic balance equations.

The current paper describes an implicit solution method for the harmonic balance discretisation of the Euler equations. Write the semi-discrete form as a system of ordinary differential equations

$$I(t) = \frac{\partial W(t)}{\partial t} + R(t) = 0, \tag{1}$$

Consider the solution W and residual R to be periodic in time and a function of ω. Some manipulation gives the following systems of equations for the Fourier series coefficients

$$\hat{R}_0 = 0 \tag{2}$$

$$\omega n \hat{W}_{b_n} + \hat{R}_{a_n} = 0 \tag{3}$$

$$-\omega n \hat{W}_{a_n} + \hat{R}_{b_n} = 0 \tag{4}$$

This is a system of $N_T = 2N_H + 1$ equations in N_T unknown harmonic terms.

This can be expressed in matrix form as

$$\omega \mathbf{A}\hat{\mathbf{W}} + \hat{\mathbf{R}} = \mathbf{0} \tag{5}$$

where A is a $N_T \times N_T$ matrix containing the entries $A(n+1, N_H+n+1) = n$ and $A(N_H+n+1, n+1) = -n$.

The problem with solving Equation 7 is in finding a relationship between $\hat{R}$ and $\hat{W}$. Hence this problem is converted back to the time domain. Split the solution into N_T discrete equally spaced sub intervals over the period $T = 2\pi/\omega$ and introduce a transformation matrix E such that

$$\hat{W} = EW \quad \text{and} \quad \hat{R} = ER.$$

Then equation 7 becomes

$$\omega AEW + ER = 0 = \omega E^{-1}AEW + E^{-1}ER = \omega DW + R$$

where $D = E^{-1}AE$ and is of the form

$$D_{i,j} = \frac{2}{N_T}\sum_{k=1}^{N_H} k \sin(2\pi k(j-i)/N_T)$$

Note that the diagonal of $D_{i,i}$ is zero.

We can then apply pseudo time marching to the harmonic balance equation. Most of the implicit CFD solver described in [3] can be reused to solve this equation. In this case the Jacobian matrix is

$$J = \begin{bmatrix} R_W|_{t_0+\Delta t} & \omega D_{1,2} & \dots & \omega D_{1,N_T} \\ \omega D_{2,1} & R_W|_{t_0+2\Delta t} & & \\ \vdots & & \ddots & \\ \omega D_{N_T,1} & \omega D_{N_T,2} & & R_W|_{t_0+T} \end{bmatrix} \tag{6}$$

where R_W is the Jacobian matrix of the ordinary CFD residual. A preconditioned Krylov solver is used to solve the sparse linear system.

Results will be shown for two test cases, namely the AGARD CT1 [4] for a pitching NACA0012 aerofoil and a pitching F-5 fighter wing with a wing-tip launcher and missile [5]. These will show that the solutions in these cases can be computed accurately using one mode, and at about a tenth of the cost of using time domain simulation.

REFERENCES

[1] Murman, S.M., A Reduced-Frequency Approach for Calculating Dynamic Derivatives, AIAA 2005-0840, 2005.

[2] Hall,K, Thomas, J and Clark, W, Computation of Unsteady Flows in Cascades Using a Harmonic Balance Technique, AIAA Journal, 40(5), 879-886, May, 2002.

[3] Badcock, K.J., Richards, B.E. and Woodgate, M.A., Elements of Computational Fluid Dynamics on Block Structured Grids using Implicit Solvers, Progress in Aerospace Sciences, vol 36, 2000, pp 351-392.

[4] Compendium of unsteady aerodynamic measurements , AGARD Report No. 702, 1982.

[5] Verification and validation data for computational unsteady aerodynamics, Research and Technology Organization (RTO), RTO-TR-26, France, Oct. 2000.