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Superconvergence and Time Evolution of Discontinuous Galerkin Finite Element Solutions

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ABSTRACT

In this talk we present our recent study on the convergence and time evolution of the error between the discontinuous Galerkin (DG) finite element solution and the exact solution for conservation laws when upwind fluxes are used. We prove that if we apply piecewise linear polynomials to a linear scalar equation, the DG solution will be superconvergent towards a particular projection of the exact solution. Thus, the error of the DG scheme will not grow for fine grids over a long time period. We give numerical examples of P^k polynomials, with $1 \leq k \leq 3$, to demonstrate the superconvergence property, as well as the long time behavior of the error. Nonlinear equations, one-dimensional systems and two-dimensional equations are numerically investigated to demonstrate that the conclusions hold true for very general cases.