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Parallel Solution Methods for Dynamic Contact Problems

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ABSTRACT

The construction of stable and efficient discretization schemes for frictional contact problems in time and space requires a careful handling of the non–smooth friction law and the non–penetration constraints at the interface. We present a new contact stabilized time discretization scheme for dynamic frictional contact problems, which is based on Rothe's Method. The resulting implicit time discretization prevents instabilities originating from the material law or from frictional effects showing up during numerical integration. In order to solve the arising large and non-linear discrete systems, we use a non-smooth multiscale method which makes the solution costs of this highly non-linear problem comparable to that of linear problems.

In addition to instabilities originating from the stiffness of the material and the friction law, also the non-penetration constraints at the contact interface have to be taken into account. As a matter of fact, classical time–discretization schemes applied to frictionless contact problems evoke oscillations in the energy and the contact forces, whose source can be found in the inequality constraints at the interface. Our new time discretization is based on the well–known Newmark scheme, but in order to avoid an energy blow–up the contact forces are treated fully implicitly [KROM99]. The contact stabilization is achieved by an additional predictor step, which guarantees a stable behavior of the contact stresses and displacements. This predictor step is realized by means of a discrete L^2–projection which takes off the artificial part of the contact forces [DKE06]. Especially for the case of Coulomb friction the stability of the contact stresses is of crucial importance as they directly influence the frictional behavior. The friction laws of Tresca and Coulomb are formulated in terms of velocities. In order to guarantee a fully implicit treatment of the contact forces in the framework of the Newmark scheme we develop a time–discretized friction potential.

Let τ denote the time step-size and $\mathbf{u}^n$ the solution at time $t^n = t^0 + n \cdot \tau$. Further, $\sigma_\nu(\mathbf{u}^{n+1})$ are the stresses in direction of the outer normal ν at the contact boundary Γ_C, and $\mathcal{F}$ is the coefficient of Friction. Then for the case of Coulomb friction we use the time–discretized friction potential

$$\eta(\mathbf{u}^{n+1}, \mathbf{v}) \quad = \quad \tau \cdot \int_{\Gamma_C} \mathcal{F} \|\sigma_\nu(\mathbf{u}^{n+1})\| \cdot \left\| \left[\frac{\mathbf{v}_T - \mathbf{u}_T^n}{\tau} \right] \right\| \, ds \,,$$

In order to formulate our new time discretization scheme, we now denote the contact forces of $\mathbf{u}^{n+1}$ by $\mathbf{F}_{\text{con}}(\mathbf{u}^{n+1})$ and the respective frictional forces by $\mathbf{F}_{\text{fri}}(\mathbf{u}^{n+1})$. Let us note, that $\mathbf{F}_{\text{con}}$ and $\mathbf{F}_{\text{fri}}$ are not

known in advance and have to be determined as part of the non-linear solution process in step (2) by solving a (quasi-)variational inequality. For some discretization in space by, e.g., finite elements with mesh-size $h > 0$, we can moreover define the discrete L^2–projection P_{L^2} onto the set $\mathcal{K}_h$ of all finite element functions fulfilling the non-penetration condition, cf. [DKE06]. This enables us to state our contact–stabilized Newmark scheme for dynamic contact with Coulomb friction:

$$\mathbf{u}^{n+1}_{\text{pred}} = P_{L^2}(\mathbf{u}^n + \tau\dot{\mathbf{u}}^n) \tag{1}$$

$$\mathbf{u}^{n+1} = \mathbf{u}^{n+1}_{\text{pred}} - \frac{\tau^2}{2}(\mathbf{F}^{1/2}(\mathbf{u}^n, \mathbf{u}^{n+1}) - \mathbf{F}_{\text{con}}(\mathbf{u}^{n+1}) - \mathbf{F}_{\text{fri}}(\mathbf{u}^{n+1})) \tag{2}$$

$$\dot{\mathbf{u}}^{n+1} = \dot{\mathbf{u}}^n - \tau(\mathbf{F}^{1/2}(\mathbf{u}^n, \mathbf{u}^{n+1}) - \mathbf{F}_{\text{con}}(\mathbf{u}^{n+1}) - \mathbf{F}_{\text{fri}}(\mathbf{u}^{n+1})) \tag{3}$$

Here, the velocities at time step n are denoted by $\dot{\mathbf{u}}^n$, the forces are given by $\mathbf{F}(\mathbf{u}^n)$, and we have set $\mathbf{F}^{1/2}(\mathbf{u}^n, \mathbf{u}^{n+1}) = \frac{1}{2}(\mathbf{F}(\mathbf{u}^n) + \mathbf{F}(\mathbf{u}^{n+1}))$.

In [KLR06], another recent approach to stabilize the contact forces is presented, which uses a modification of the discrete mass by removing the mass from the boundary. As this method requires a discretization first in space then in time, no remeshing in space would be possible. However, since the detection of the contact boundary is part of the solution process, we are interested in adaptive refinement in space. With respect to the formulation of our method it can be seen, that it allows for different meshes at different time steps.

For the case of Tresca friction we have to solve a variational inclusion in each time step in step (2) and for the case of Coulomb friction we get a quasi–variational inclusion. In the fully discrete case both kinds of variational inclusions are uniquely solvable. We employ a parallel and non–smooth multiscale method for the solution of the arising (quasi–)variational inequalities [Kra07]. This method allows for the solution of frictional contact problems with optimal complexity. On the finest level, a non-linear relaxation method accounts for the firctional effects. Constrained and non-linear coarse grid corrections then assure the monotonicity and the efficiency of the multigrid method. The efficiency and flexibility of our resulting solution method for dynamic frictional contact problems is demonstrated along different examples from mechanics. As an outlook, we moreover consider the case of large deformations and present a new approach for the representation and discretization of the kinematical non–penetration condition by means of implicit surfaces.

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