

## A multiscale method for weakly coupled non-harmonic oscillators

\*Gil Ariel<sup>1</sup>, Bjorn Engquist<sup>2</sup> and Richard Tsai<sup>3</sup>

<sup>1</sup> The University of Texas at  
Austin  
1 Univ. Sta C1200, Austin,  
TX78712, USA  
ariel@math.utexas.edu

<sup>2</sup> The University of Texas at  
Austin  
1 Univ. Sta C1200, Austin,  
TX78712, USA  
[engquist@ices.utexas.edu](mailto:engquist@ices.utexas.edu)

<sup>3</sup> The University of Texas at  
Austin  
1 Univ. Sta C1200, Austin,  
TX78712, USA  
[ytsai@ices.utexas.edu](mailto:ytsai@ices.utexas.edu)

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### ABSTRACT

A multiscale method for computing the effective slow behavior of a system of weakly coupled non-harmonic oscillators is presented. The oscillators may be either in the form of a periodic solution or a stable limit cycle. Furthermore, the oscillators may be in resonance with one another and thereby generate some hidden slow dynamics. The proposed method relies on correctly tracking a set of slow variables whose dynamics is closed up to a small perturbation, and is sufficient to approximate any variable and functional that are slow under the dynamics of the ODE. The advantages of the method is demonstrated with a few examples. Particular emphasis is given to the effect of synchronization. Harmonic oscillators with slowly varying properties are also studied. The algorithm follows the framework of the heterogeneous multiscale method.

### REFERENCES

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