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STATISTICAL VOLUME ELEMENT METHOD FOR PREDICTING PROBABILISTIC MATERIAL CONSTITUTIVE RELATIONS

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ABSTRACT

The inherent random nature of a heterogeneous microstructure in materials has a direct influence on the material constitutive response which is critical for predicting material behavior and designing reliable products. Random microstructure configurations introduce variations in effective macroscale material constitutive properties [1-3]. There is an inevitable need to establish multi-length scale probabilistic microstructure-constitutive property relations in materials design.

We have developed a predictive statistical volume element method to analyze, quantify, and calibrate probabilistic microstructure-constitutive property relations by statistical means. Statistical volume element simulations are conducted to predict material constitutive properties corresponding to various realizations of random material microstructure configurations. A computing framework has been developed to link random configuration generators and finite element analysis. The proposed approach is applied to examine a porous steel alloy material for demonstrative purposes.

To analyze the influence of random material microstructure on material constitutive properties, a statistical cause-effect analysis approach is proposed to provide an importance ranking of microstructure parameters. Within the proposed approach, statistically significant microstructure parameters are first identified based on their linear impacts on material constitutive properties [4]. Global sensitivity analysis [5] is then carried out to provide a more comprehensive importance ranking of these critical microstructure parameters considering both main and interaction effects.

Based on an ensemble of statistical volume element simulations, the uncertainties in the

material constitutive relation are quantified in terms of distribution, statistical moments and correlation. The obtained probabilistic constitutive relations are used to calibrate the model parameters in constitutive relation models by applying a statistical calibration process. Calibration of the Bammann-Chiesa-Johnson (BCJ) constitutive model [6] is employed as an example to illustrate the statistical calibration process. The calibrated material constitutive models that incorporate the uncertainties propagated from random material microstructure can be used in a coarser scale simulation in a multiscale design and analysis content.

Our proposed techniques are generic enough to be applied to more sophisticated multiscale material models in either a hierarchical or a concurrent manner. The capability of deriving probabilistic material constitutive relations is essential in model validation process where the statistical computational results will be compared against random experimental results following the same statistical model calibration procedure. The capability will also allow designers to assess the reliability of product performance by introducing the statistical representation of material constitutive relations.

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