8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Comparison of two FFT-BIE methods for Poisson-type equation

***Xuefei He[1], Kian-Meng Lim[1,2] and Siak-Piang Lim[1,2]**

[1] Singapore-MIT Alliance
E4-04-10, 4 Engineering Drive 3
Singapore 117576

[2] Department of Mechanical Engineering
National University of Singapore
9 Engineering Drive 1, Singapore 119260

E-mail: X.F. He (smahxf@nus.edu.sg), K.M. Lim (limkm@nus.edu.sg),
S.P. Lim (mpelimsp@nus.edu.sg)

Key Words: *boundary element method (BEM), Poisson-type equation, non-linear equation, fast Fourier transform (FFT).*

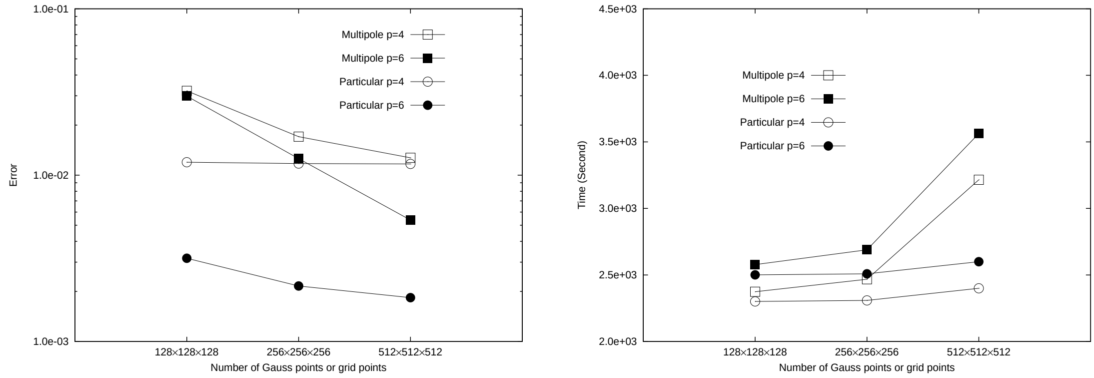
ABSTRACT

The boundary integral equation (BIE) method has the advantage of transforming the governing differential equation to the boundary, and requiring only the discretization of the boundary of the computational domain (such as for the Laplace equation). This simplifies the mesh generation (especially for 3D problems), and also reduces the number of unknowns in the problem. However, for Poisson-type equations, we need handle a domain integral corresponding to the non-homogeneous term, especially in cases when it cannot be transformed to the boundary. Many techniques have been proposed to handle this domain integral in relation to the BIE method [1]. In the recent years, fast solvers had also been developed for the BIE method [2,3].

In this paper, we study two fast algorithms for handling the domain integral in the Poisson equation, and combine these algorithms with a fast solver for the Laplace equation based on fast Fourier transform (FFT) on multipoles. The first fast algorithm uses a FFT-multipole scheme to accelerate the domain integral, similar to the use of FFT and multipoles in handling the boundary sources in the Laplace equation. The second fast algorithm separates the solution of the Poisson equation into a particular solution and a homogeneous solution. The particular solution is evaluated rapidly using FFT, and the homogeneous solution is obtained using the FFT on multipoles fast solver for Laplace equation.

We consider the Poisson equation $\nabla^2 u(\mathbf{x}) = 1$ subjected to Dirichlet boundary condition $u(\mathbf{x}) = \frac{x_1^2}{2}$ on the boundary of a sphere. Figure 1 shows the accuracy of the numerical solutions compared with the analytical solution given by $\frac{\partial u}{\partial \mathbf{n}} = x_1 n_1$, and the computational time taken by each method. The FFT accelerated particular solution method (Particular) is shown to be faster and more accurate than the FFT-multipole accelerated domain integration method (Multipole). For each method, two sets of results are obtained using two different orders of multipole expansion, $p = 4$ and 6, for the FFT on multipole solver.

As the FFT accelerated particular solution method is found to be superior in terms of computational efficiency, it is further extended to solve non-linear Poisson-type equations. The Richardson iterative



(a) Accuracy (b) Total time

Figure 1: Results for the Poisson equation with 33884 nodes

scheme is used, and a Poisson equation is solved within each iterative step. We consider the non-linear equation given by $\nabla^2 u(\mathbf{x}) = u + u^3$ and subjected to Dirichlet boundary condition $u(\mathbf{x}) = \tan(\frac{x_1+x_2+x_3}{\sqrt{6}})$ on the boundary of a sphere. The analytical solution is given by $\frac{\partial u^*}{\partial \mathbf{n}} = \frac{1+u^2}{\sqrt{6}}(n_1 + n_2 + n_3)$. Table 1 shows the accuracy of the numerical solutions and the computational time taken for various sizes of boundary mesh used. It is found that the iterative solver typically takes four iterations to converge to an accuracy of 1% or less. The computational time needed scales as $O(N)$ approximately, where N is the number of nodes.

Table 1: Numerical results for the non-linear equation

Number of nodes	Computational time (Second)		Error	
	$p = 4$	$p = 6$	$p = 4$	$p = 6$
4858	3953	4851	0.70%	0.30%
8566	3989	4914	0.71%	0.24%
19234	7557	8035	0.90%	0.21%
33884	14188	15661	1.0%	0.21%

The fast algorithm presented in this paper that combines the FFT accelerated particular solution and FFT on multipole solver for the homogeneous solution is shown to provide an efficient way of solving Poisson-type equations. It has a great potential in application to solving large scale problems (possibly with more than 100,000 degrees of freedom), and non-linear equations which involve many iterative steps.

REFERENCES

[1] M.S. Ingber, A.A. Mammoli and M.J. Brown. "A comparison of domain integral evaluation techniques for boundary element methods". *Int. J. Numer. Meth. Eng.*, Vol. **52**, 417-432, 2001.

[2] L. Greengard and V. Rokhlin. "A fast algorithm for particle simulations". *J. Comput. Phys.*, Vol. **73**, 325-348, 1987.

[3] K.M. Lim, X.F. He and S.P. Lim. "Fast Fourier transform on multipoles (FFTM) algorithm for Laplace equation with direct and indirect boundary element method". *Comput. Mech.*, Vol. **41**, 313–323, 2008.