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Homogeneous Variational Integrators for Lagrangian Dynamics on Two-Spheres

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ABSTRACT

Homogeneous variational integrators for Lagrangian flows on two-spheres are constructed by lifting the variational principle on S^2 to a constrained variational principle on $SO(3)$, through the use of constrained variations which quotient out the local isotropy subgroup of the action of $SO(3)$ on S^2. While the construction relies on a choice of a connection that defines a complementary horizontal direction to the isotropy group direction, the discrete flow is independent of the choice of connection.

The evolution equations for a system of coupled objects whose configuration space is given by $(S^2)^n$ can be compactly expressed as follows,

$$M_{ii}\ddot{q}_i = q_i \times (q_i \times \sum_{\substack{j=1\\ j\neq i}}^{n} M_{ij}\ddot{q}_j) - (\dot{q}_i \cdot \dot{q}_i)M_{ii}q_i + q_i \times \left(q_i \times \frac{\partial V}{\partial q_i}\right)$$

These are global, singularity free expressions in continuous time that have the following discrete time analogue,

$$\frac{1}{h}(q_{i_k} \times \sum_{j=1}^{n} M_{ij}(-q_{j_{k+1}} + 2q_{j_k} - q_{j_{k-1}})) - hq_{i_k} \times \frac{\partial V_k}{\partial q_{i_k}} = 0.$$

The corresponding discrete flow is symplectic, momentum preserving, and exhibits exceptional energy stability, while automatically preserving the unit length constraints without the need for constraints, reprojection, or local coordinates. The second-order accurate version of the resulting algorithm can be viewed as an index reduced version of the RATTLE algorithm, but the construction can be generalized to an arbitrary order of accuracy.

These numerical methods provide the basis for constructing geometrically exact numerical schemes for representing multi-jointed systems, flexible structures, and surfaces arising in modern computational mechanics applications.