

A Global Explicit Residual Based Error Estimator for the eXtended Finite Element Method in *Code_Aster*

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ABSTRACT

For its engineering related to their plants, Électricité de France develops a computational software *Code_Aster*¹ based on the finite element method. In the case of computation of industrial cracked structures, the traditional finite element method requires the creation of a mesh that respects the geometry of the crack and a remeshing at each step of the crack propagation. At this time, the creation of 3D meshes for complex structures is extremely expensive and the robustness is not perfect.

The eXtended Finite Element Method (XFEM)¹ provides the possibility to insert, in a finite element mesh, a surface of discontinuity without changing the mesh and this, whatever the crack position. The idea is to enrich the traditional shape functions basis with discontinuous functions. Thus, with the XFEM, many cases (various positions and crack shapes) can be studied on the same mesh. During the crack propagation, the mesh is preserved whereas it must be recreated for each situation with the finite element method. The XFEM has been implemented in *Code_Aster* with contact taken into account on the crack faces³.

The accuracy of a finite element approximation is generally evaluated with the methods of discretization error estimation^{4,5}. Error estimators are used to locate zones with error in order to use procedures of mesh adaptation to improve and optimize the global quality of a computation for a given criteria (computing time, number of element, etc.). A type of local estimator, known as in quantity of interest or goal oriented⁶, provides an error in a zone on a precise mechanical quantity (Von Mises stress, stress intensity factor, etc.).

The alternative presented by XFEM is very attractive from the point of view of mesh management. But to improve results quality, fine meshes are necessary around the crack front. The use of error estimation methods coupled to mesh adaptation procedures becomes necessary.

To estimate the discretization error for XFEM, several estimators were proposed, based on

a smooth strain field^{7,8} (like in the Zienkiewicz-Zhu a posteriori error estimator) or the residual⁹ (like in the Babuška-Rheinboldt estimator). An estimator based on the residual, developed by Lleras and Hild, has been implemented in Code_Aster. Numerical tests show the good efficiency of the estimator and particularly that the approximate error converges to the exact error. For the simulation of industrial structures, the estimator is used as mesh refinement criteria in an adaptive process, driven by HOMARD¹⁰, a 3D mesh refinement and coarsening software developed by Électricité de France. We will try to develop a mesh optimality criteria that minimize the number of elements for a given accuracy. Our final goal is to develop and to implement in Code_Aster, a goal-oriented residual based error estimator following the ideas developed by Delmas *et al.*¹¹.

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