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Adaptive sparse polynomial chaos expansions using a sequential experimental design

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ABSTRACT

Polynomial chaos (PC) expansions [1,2] allow the analyst to represent explicitly the stochastic response of a mechanical model whose input parameters are random variables and/or random fields. It has been shown in [3,4] that the PC coefficients can be efficiently computed using a non intrusive regression approach. However, the required number of model evaluations (i.e. the computational cost) increases with the PC size which itself dramatically increases with the number of input variables when the common truncation scheme of the PC expansion is applied. For a large class of mechanical problems though, many PC terms may be discarded since high order interactions are negligible and since all the input variables do not have the same influence on the model response.

In this paper, an iterative procedure is proposed in order to build a *sparse* PC representation of the random response. First, an initial Latin Hypercube design (LHD) is generated and the corresponding model evaluations are performed. Then a PC approximation of the response is built by iteratively increasing its total degree as well as the interaction order of its basis polynomials. At each step, candidate terms are added in turn to the metamodel and one computes the induced increase of the determination coefficient R^2. One eventually retains those polynomials that lead to a meaningful raise of this statistics, i.e. greater than a given cut-off value.

Beside this adaptivity in terms of PC basis, the experimental design is systematically completed prior to performing regression so that the information matrix is well-conditioned. This is achieved by using an original sampling scheme, namely the *Nested Latin Hypercube Design* (NLHD) technique, which is inspired from a method described in [5]. NLHD is aimed at building experimental designs from an initial LHD by recycling all the previous samples and by adding points such that the new set has almost a LHD structure.

The approximation error of the various PC approximations is assessed by the empirical lack of fit and a resampling-based estimate, namely the leave-one-out error (see e.g. [6]). A rather small number of PC terms are eventually retained (sparse representation), which may be computed by means of a low number of model evaluations compared to a full PC approximation.

The method is illustrated by the moment, sensitivity and reliability analyses of linear elastic structural problems which involve more than 20 input random variables.

REFERENCES

[1] R. Ghanem and P. Spanos. *Stochastic Finite Elements : A Spectral Appoach*. Courier Dover Publications, 2003.

[2] C. Soize and R. Ghanem. Physical systems with random uncertainties: chaos representations with arbitrary probability measure. *SIAM J. Sci. Comput.*, 26(2):395-410, 2004.

[3] M. Berveiller. *Stochastic finite elements: intrusive and non intrusive methods for reliability analysis*. PhD thesis, Université Blaise Pascal, Clermont-Ferrand, 2005.

[4] M. Berveiller, B. Sudret and M. Lemaire. Stochastic Finite Elements: a non intrusive approach by regression. *Revue Européenne de Mécanique Numérique*, 15(1,2,3):81-92, 2006.

[5] G.G. Wang. Adaptive Response Surface Method Using Inherited Latin Hypercube Design Points. *J. Mech. Design*, 125:210-220, 2003.

[6] O. Bousquet and A. Elisseeff. Stability and Generalization. *J. Mach. Learn. Res.*, 2:499-526, 2002.