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Special Hybrid Multilayer Finite Elements for 3-D Stress Analyses Around Holes in Laminated Composites

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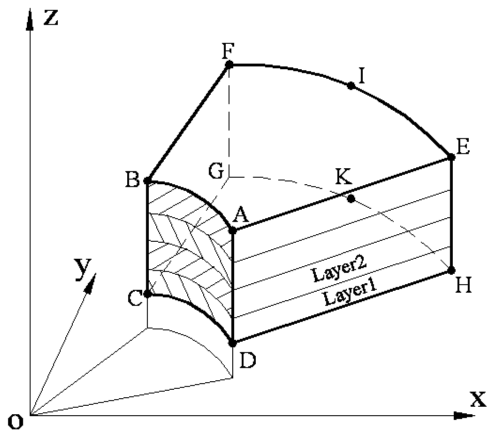
ABSTRACT

A new kind of 3-dimensional assumed stress hybrid multilayer element with a traction-free cylindrical surface ABCD (Figure 1) is derived by using an extended Hellinger-Reissner principle which can be obtained by introducing Lagrange multipliers to release the continuity of interelement displacement.

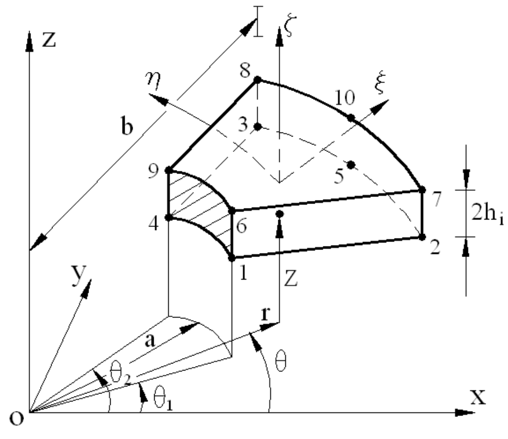
The energy functional for multilayer element is expressed as

$$\Pi_{mR}(\boldsymbol{\sigma},\boldsymbol{u}_q,\boldsymbol{u}_\lambda)=\sum_n\sum_j\left\{\int_{V_{nj}}[-\frac{1}{2}\boldsymbol{\sigma}^{(j)\mathrm{T}}\mathbf{S}^{(j)}\boldsymbol{\sigma}^{(j)}+\boldsymbol{\sigma}^{(j)\mathrm{T}}(\boldsymbol{D}\boldsymbol{u}_q^{(j)})\right.$$
$$\left.-(\boldsymbol{D}^{\mathrm{T}}\boldsymbol{\sigma}^{(j)})^{\mathrm{T}}\boldsymbol{u}_\lambda^{(j)}-\overline{\mathbf{F}}^{(j)\mathrm{T}}(\boldsymbol{u}_\lambda^{(j)}+\boldsymbol{u}_q^{(j)})]dV-\int_{S_{\sigma_{nj}}}\overline{\mathbf{T}}^{(j)\mathrm{T}}\boldsymbol{u}_q^{(j)}dS\right\} \qquad (1)$$

where $\boldsymbol{\sigma}^{(j)}$: stresses of layer *j*; $\boldsymbol{u}_q^{(j)}$: element displacement of layer *j*; $\boldsymbol{u}_\lambda^{(j)}$: element internal displacement of layer *j*; $\boldsymbol{D}$: differential operator; $\mathbf{S}$: elasticity material property matrix; V_{nj}: volume of layer *j* for element *n*; $\overline{\mathbf{F}}^{(j)}$: body forces and $\overline{\mathbf{T}}^{(j)}$: prescribed tractions of layer *j*.



(a) Geometry of the element



(b) One layer of the element

Figure 1: Geometry of the special element with a traction-free cylindrical surface

The essential terms of the assumed stresses are complete polynomials in the natural coordinates of each layer. The equilibrium conditions are imposed a prior in a variational sense through the internal displacements which are also expanded in the natural coordinates. The continuity of the transverse stresses on the interlayer surface and the traction-free boundary conditions over the cylindrical surface are satisfied exactly, while the interelement traction continuity is imposed in a variational sense a posterior.

All components of displacement are included since bending/stretching coupling may occur. The transverse-shear deformation effects are incorporated in each layer with displacement continuity enforced along interlayer surface.

The 3-dimensional distributions of stress around a central quasi-elliptic hole in a cross ply laminate under tension are analyzed by using the combination of two kinds of three-dimensional special hybrid stress multilayer finite elements. One kind is present special finite element which contains a traction-free cylindrical surface and the other special finite element contains a traction-free planar surface which is derived based on a modified complementary energy principle.

Examples have indicated that the combination of these special multilayer finite elements is far superior in predicting the distributions of circumferential stresses and transverse stresses for laminated composites with holes, when very coarse finite meshes are used near the free-edge of cutout. Meanwhile the method is computationally much more efficient than the ordinary assumed stresses finite elements and the conventional assumed displacement elements.

The special finite elements are applicable to efficient analyses of 3-dimensional stresses around other kinds of cutouts in thin to thick laminated plates — such as rectangular holes, eccentric holes, U-shaped grooves etc.. These elements are also currently extended to free-edge effects of laminated shell.

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