

## Parallel Hierarchical Optimization of Geometrically Nonlinear Laminated Composite Structures

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### ABSTRACT

The solution of large scale optimization problems involving three dimensional laminated composite structures is a challenge, which requires usage of modern state of the art computer architectures and the development of efficient tools for analysis and design. The general composite structural design problem may involve complicating factors such as many material layers, complicated layups, geometrically nonlinear response under loading, and three dimensional effects.

A general laminated composite plate/shell/solid structure may fail in a wide variety of failure modes such as tensile failure, delamination failure, etc. To take into account all these different failure effects, it is a necessity to utilize analysis and optimization tools which are based on fully three dimensional kinematics and constitutive relations. The usage of fully three dimensional analysis tools makes it possible to efficiently address a wide range of problems which cannot be solved using simpler analysis and optimization tools.

In this work it is sought to solve the problem of maximizing the safety against failure of a fully three dimensional laminated composite shell structure, taking into account failure effects using three dimensionally stress-based failure criteria. The stress criteria used in this work are the Tsai-Wu ([1]) and Ye ([2]) stress criteria for predicting in-plane and transverse delamination failure. The fibre angles of the laminated composite structure are chosen as design variables, as identifying optimal laminate layup is a complicated task involving efficient tools for synthesis.

As a framework of analysis, two-step hierarchical finite element models of the laminated composite structure are developed and solved. In step one, a coarse mesh finite element model of the laminated structure is developed. The coarse mesh finite element model consists of locking-free eight node equivalent single layer solid shell finite elements which are based on a continuum mechanics formulation developed in [3]. Based on the solution of the coarse finite element analysis problem, a refined analysis/optimization model is automatically generated by refining the finite element model through the thickness in localized "zones of interest". This refinement procedure makes it possible to obtain detailed information about the through-the-thickness stress/strain variations in the structure while maintaining a relatively low total number of degrees of freedom in the computational model. Enforcement of displacement continuity across the mesh-refinement interfaces is performed through the method of Lagrange multipliers. The new finite element model with high resolution in zones of interest is used as offset for the solution of the corresponding high resolution geometrically nonlinear analysis/optimization

problem.

The optimization problem is formulated as a mathematical programming problem and solved through use of efficient gradient based techniques, where expressions for the design sensitivities are developed through the direct differentiation approach applied to the governing system of equations for the converged analysis problem. The sensitivities of the local failure functions with respect to the fibre angles are then recovered through a Taylor expansion. The sensitivities are calculated through the semi-analytical approach. The layered solid shell finite elements are based on a mixed variational formulation, and the design sensitivity analysis thus involves the development of sensitivity expressions for mixed finite elements. To decrease the number of sensitivity evaluations necessary to solve the optimization problem an active set strategy is employed to identify the elements critical to the solution of the optimization problem. The optimization problem is solved by the Sequential Linear Programming method.

The solution of geometrically nonlinear three dimensional composite optimization problems is a demanding problem which requires the usage of efficient computer algorithms. In this work optimized mathematical libraries have been utilized in conjunction with efficient multi-threaded Direct Sparse Solver routines, resulting in the efficient solution of large linear systems of equations on Symmetric MultiProcessing (SMP) computer architectures. To further exploit the advantages of modern SMP computer architectures, Open Multi-Processing [4] (OpenMP) is employed to obtain a high-performance optimization algorithm by multi-threading all routines involving element routines in the fine mesh problem. A Message Passing Interface [5] (MPI) is used in conjunction with multi-threading to develop a framework for the parallel solution of optimization problems involving three dimensional laminated composite structures on a Cluster of SMP's.

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