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BOUNDARY ELEMENT METHOD SOLUTION IN THE CASE OF CONCENTRATED FORCES AND MOMENTS

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ABSTRACT

Until now the Boundary Element Method (BEM) has been widely used for obtaining numerical solutions in many problems in science and engineering [1]. But, problems with mathematical singularities appear in the solution of boundary value problem independently of its nature, due to the solution approach. The solution of a large class of boundary value problems in physics and engineering [1-7], can be reduced to Boundary Integral Equations (B.I.Es). The integral identity for the displacements u_i at points on the surface, S, with unit outward normal $n_j\,(j=1,2,3)$, of a three dimensional body, Ω, in the absence of body forces, is given by [1,4,5]:

$$d(y)u_i(y)+\int_S u_\ell(x)\,S^*_{\ell i}(x,y)\,dS(x)=\int_S t_\ell(x)\,D^*_{\ell i}(x,y)\,dS(x) \tag{1}$$

where, $u(x)=\{u_1(x),u_2(x),u_3(x)\}^{\mathrm{T}}$, the displacements field,

$t(x)=\{t_1(x),t_2(x),t_3(x)\}^{\mathrm{T}}$ the traction vector applied to the surface, S,

and $d(y)$ is a free term that depends in the shape of the boundary [1,4].

The integrals in equation (1) are in the sense of Cauchy principal value. The tensors $S^*_{\ell i}(x,y)$ and $D^*_{\ell i}(x,y)$ are known fundamental displacement and traction solutions respectively for a point force in an infinite domain [1,4].

The solution of B.I.Es can be performed numerically. Up to now many methods [4-7] which can confront the Cauchy type singularity of the kernel, have been proposed. Unfortunately less attention has been paid to some inmost singularities of B.I.Es arising from concentrated forces or moments, and influencing the solution and the convergence of their numerical solution.

The concentrated forces or moments cause corresponding singularities in the right – hand side function and in the unknown function of the Boundary Integral Equation. These singularities need a special treatment in the process of integration. Ignoring these singularities can be catastrophic for the solution in the vicinity of these singularities and it has repercussions in the accuracy of the whole solution. In this paper we will try to illuminate and classify these singularities by presenting the cause of their appearance and to confront them by proposing appropriate quadrature rules.

REFERENCES

[1] C.A. Brebbia, and S. Walker, *Boundary Element Techniques in Engineering*, Butterworths, London, 1980

[2] P.S. Theocaris, and G. Tsamasphyros, "On the solution of boundary-value problems in the plane elasticity for multiply-connected region (I: The second boundary-value problem)", *Lett. Appl. Eng.* Vol. **3**, pp. 167-176, (1975).

[3] E.E. Theotokoglou, and G. Tsamasphyros, "An integral equation solution for cracked half-planes bonded together and containing debondings along their interface", *International Journal of Fracture*, Vol. **55**, pp. 1-16, (1992).

[4] V. Sladek, and J. Sladek, *Singular Integrals in the Boundary Element Methods*, Computational Mechanics Publications, Southamnpton, UK, 1998.

[5] E.E. Theotokoglou, and G. Tsamasphyros, "Derivation of exact expressions for two-dimensional singular and finite-part integrals applicable in solid mechanics", *Engineering Analysis with Boundary Elements*, Vol. **22**, pp.125-132, (1998).

[6] Y.-S. Chan, A.C. Fannjiang and G.H. Paulino, "Integral equations with hypersingular kernels-theory and applications to fracture mechanics", *Int. J. Engineering Science*, Vol. **41**, pp. 683-720, (2003).

[7] G. Tsamasphyros and E.E. Theotokoglou, "A quadrature formula for integrals with nearby singularities", *Int. J. Numerical Methods in Engineering*, Vol. **67**, pp. 1082-1093, (2006).