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ON SOME INVERSE PROBLEMS OF PARAMETER IDENTIFICATION IN LINEAR ELASTICITY

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ABSTRACT

In this talk we deal with inverse problems of identifying material parameters in linear elasticity from measured deformations of test objects. Our focus is on two-dimensional objects with non-trivial (L-shaped, non-convex) geometry and on various experimental design situations (Dirichlet and Neumann data on different parts of the boundary). We consider questions of identifiability, stability, robustness and numerical efficiency of identification problems occurring in this field.

For example, we consider the identification of the Lamé constants λ and μ, which are defined as

$$\lambda := \frac{E\nu}{1-\nu^2} \qquad \text{and} \qquad \mu := \frac{E}{2(1+\nu)},$$

where E denotes the modulus of elasticity and ν the Poisson's ratio. Here small deformations and a linear elastic material behavior with plain stress are assumed. Using the material tensor $C(\lambda, \mu)$ the relation between stress $\sigma(u)$ and strain $\varepsilon(u)$ is given by the linear deformation law

$$\sigma(u) = C(\lambda,\mu)\varepsilon(u) \qquad \text{with} \qquad \varepsilon_{ij} := \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad i,j = 1,2\ .$$

Then the observed deformation u for a given parameter couple $p = (\lambda, \mu)$ arises as the solution of the partial differential equation boundary value problem

$$\begin{aligned} \operatorname{div}\sigma(u) &= 0 && \text{on } \Omega, \\ u &= g_D && \text{on } \Gamma_D, \\ \sigma(u)\cdot\vec{n} &= g_N && \text{on } \Gamma_N, \end{aligned} \tag{1}$$

where Ω characterizes the two-dimensional domain and Γ its boundary with Dirichlet conditions on the part Γ_D and Neumann conditions on the part Γ_N.

For this model and the corresponding inverse problem the associated forward operator transforms the given parameter couple p to the uniquely determined weak solution of the P.D.E. problem (1). While solving the inverse problem we have to identify p from a given data vector $\underline{u}_{data}$, which is yielded by measured discrete deformations of the object. For the solution of the inverse identification problem unregularized Gauß-Newton methods and multi-parameter regularization approaches (see, e.g., [1]) are applied.

In the first part of the presentation we shortly introduce some basics of the analysis of linear elasticity with plain stress. The forward operator is defined and derivatives are considered. The second part gives a short overview of finite-elements discretization that is used for numerical solution of the problem. Then we formulate algorithms for solving the inverse problem. Several strategies will be applied. An unregularized least-squares minimization using the Gauß-Newton method is compared with multi-parameter regularization approaches. For solving the optimization problems arising from multi-parameter methods we introduce Lagrange- and SQP-techniques (see [2] for such techniques and [3] for implementation). In the last part extended numerical studies on a test problem are presented that illustrate the numerical treatment of the problem and different experimental design situations.

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