8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

IDENTIFICATION OF RELAXATION TIME IN BIO-HEAT TRANSFER EQUATION

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Key Words: *Inverse Problems, Cattaneo-Vernotte Equation, Numerical Modelling*

ABSTRACT

According to the newest opinions the heat conduction proceeding in the biological tissue domain should be described by the hyperbolic equation (Cattaneo and Vernotte equation [1]) in order to take into account its nonhomogeneous inner structure. So the following bio-heat transfer equation is considered

$$c\left[\tau\frac{\partial^2 T(x,\,t)}{\partial t^2}+\frac{\partial T(x,\,t)}{\partial t}\right]=\lambda\nabla^2 T(x,\,t)+Q(x,\,t)+\tau\frac{\partial Q(x,\,t)}{\partial t} \tag{1}$$

where c, λ denote the volumetric specific heat and thermal conductivity of tissue, $Q(x, t)$ is the capacity of internal heat sources due to metabolism and blood perfusion, τ is the relaxation time (for biological tissue is a value from the scope 20-35 s), T is the tissue temperature, x, t denote the spatial co-ordinates and time. The function $Q(x, t)$ is equal to

$$Q(x,\,t)=G_B c_B\left[T_B-T(x,\,t)\right]+Q_m \tag{2}$$

where G_B is the blood perfusion rate, c_B is the volumetric specific heat of blood, T_B is the artery temperature and Q_m is the metabolic heat source. It should be pointed out that for $\tau = 0$ the equation (1) reduces to the well-known Pennes bioheat equation.

The equation (1) is supplemented by the boundary conditions

$$\begin{aligned} x\in\Gamma_1: &\quad T(x,\,t)=T_b(x)\\ x\in\Gamma_2: &\quad q(x,\,t+\tau)=-\lambda\mathbf{n}\cdot\nabla T(x,\,t)=q_b(x)\end{aligned} \tag{3}$$

and initial ones

$$t=0:\quad T(x,\,t)=T_0,\quad \left.\frac{\partial T(x,\,t)}{\partial t}\right|_{t=0}=0 \tag{4}$$

where Γ_1, Γ_2 are the surfaces limiting the domain, $q(x, t+\tau)$ is the boundary heat flux, $T_b(x)$, $q_b(x)$ are known boundary temperature and boundary heat flux and T_0 is known initial temperature of biological tissue.

The inverse problem considered here consists in the identification of τ on the basis of heating curves at the points selected from tissue domain (the testing computations show

that the information obtained using one sensor is quite sufficient). In order to generate the visible temperature gradients in the domain analyzed, the skin surface is subjected to the external heat flux which causes the increase of boundary temperature (equation (3a)).

To solve the inverse problem formulated the least square criterion in the form [2]

$$S(\tau) = \sum_{f=1}^{F} \left(T^f - T_d^f \right)^2 \tag{5}$$

is applied. In equation (5) T_d^f is the 'measured' temperature at the point x_i for time t^f, while T^f is the calculated temperature for the set of parameters assumed. The necessary condition of functional (5) minimum leads to the equation which allows to find the optimal value of τ using the gradient method. The final equation contains the sensitivity coefficients. They are determined using the direct approach of sensitivity analysis.

Both the basic problem and the sensitivity one have been solved using the dual reciprocity boundary element method [3]. On the stage of numerical computations the 1D task has been solved (the layer of tissue (G=1cm) limited by the skin surface and conventionally assumed internal one). The following input data have been introduced λ = 0.5 W/mK, c = 4.2 MJ/m^{3}K, c_B = 3.9962, G_B = 0.002 1/s, T_B =37 °C, Q_m = 420 W/m^3 , τ = 35 s (this value is identified), T_0 = 37 °C. The boundary condition on the skin surface has been assumed in the form $T_b(t)$ = 37+0.25t, $t \leq 32$, position of sensor corresponds to x = 0.001 m. In Figure 1 the course of identification process is shown. One can see that the real value of τ has been obtained after 4 iterations.

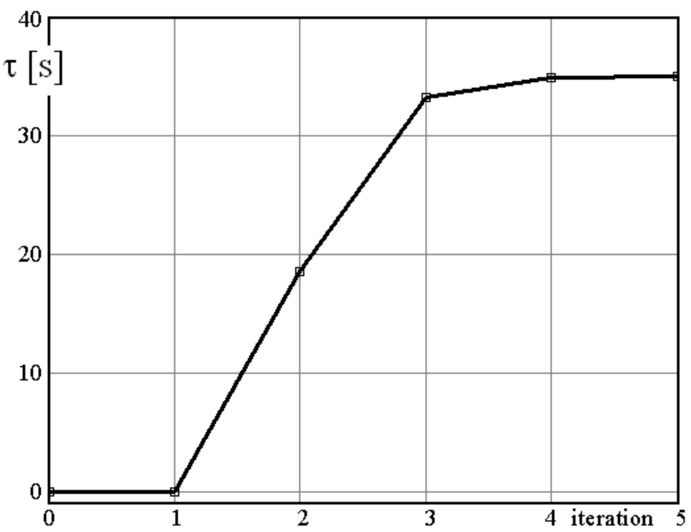

Fig. 1. Identification of τ

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