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COMPARISON OF RECENTLY DEVELOPED RECOVERY-TYPE DISCRETIZATION ERROR ESTIMATORS FOR THE EXTENDED FINITE ELEMENT METHOD

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ABSTRACT

The extended finite element method (XFEM) has recently emerged as a highly efficient numerical method for crack modelling. The XFEM uses the partition of unity method to model cracks, adding new degrees of freedom to introduce the discontinuity of the displacement field across the faces of the crack and to represent the asymptotic displacement field around the crack tip. Thanks to the advances made in the XFEM in recent years, the method is now considered to be a robust and highly accurate means of analyzing fracture problems. Nonetheless, like the FEM, the XFEM also yields results that are affected by the so-called discretization error. The importance of error estimation in numerical analysis is widely acknowledged. Because of the increasing importance and use of partition of unity (PUM) based generalized finite element methods, it is particularly important to develop procedures that are capable of providing accurate error estimates for these methods, in particular because they tend to use coarse discretizations.

The error assessment tools used in finite element analysis are well known and are usually classified into two families: residual type error estimators and recovery based error estimators. The former are based on the ideas of Zienkiewicz and Zhu [1] and [2] are often preferred by practitioners because they are robust and simple to use.

The literature on error estimation methods for mesh based partition of unity methods, however, is very limited. Two research groups have recently developed accurate error estimators specifically adapted to the XFEM framework:

- On the one hand, Bordas *et al*. [3] and Bordas and Duflot [4] have presented a

recovery based error estimator for XFEM. This method, called extended moving leas squares (XMLS), proposes to enrich intrinsically the Moving Least Square recovery to include information about the near tip fields, and uses the diffraction method to introduce the discontinuity in the recovered fields. This method provides accurate results with effectivity indices of the error estimator close to unity (optimal value) for 2D and 3D fracture mechanics problems. Duflot and Bordas [5] propose a global recovery technique where the recovered solution is sought in a space spanned by the near tip strain fields obtained from differentiating the Westergaard asymptotic expansion, although the results provided by this technique are not as accurate as those in [3] and [4], they require less computational power.

- On the other hand Ródenas *et al*. [6] have developed an adaptation of the Superconvergent Patch Recovery (SPR) technique for the XFEM framework, called SPR_{XFEM}, which also provides accurate error estimations. This adaptation is based on 3 fundamental aspects: *a)* the use of a *singular* + *smooth* stress field decomposition technique involving the use of different recovery methods for each field: standard SPR for the smooth field, and reconstruction of the recovered singular field using the stress intensity factor *K* for the singular field; *b)* direct calculation of recovered stresses at integration points using conjoint polynomial enhancement; and *c)* assembly of patches with elements intersected by the crack using different stress interpolation polynomials along each side of the crack.

This paper is a collaboration between these two research groups with the objective of presenting a comparison for 2D problems between the XMLS technique and an enhanced version of the SPR_{XFEM} technique. Sequences of uniformly refined meshes for problems with known exact solution have been analyzed in order to evaluate accuracy of the error estimators, effectivity, robustness… Advantages and disadvantages of each method are also provided in the comparisons.

REFERENCES

[1] O.C. Zienkiewicz, J.Z. Zhu. “A simple error estimation and adaptive procedure for practical engineering analysis”. *Int J Num Meth Eng*, Vol. **24**, pp. 337-357, (1987).

[2] O.C. Zienkiewicz, J.Z. Zhu. “The superconvergent patch recovery and a posteriori error estimates. Part I: The recovery technique”. *Int J Num Meth Eng*, Vol. **33**, pp. 1331-1364, (1992).

[3] S. Bordas, M. Duflot, P. Le. “A simple error estimator for extended finite elements”. *Com Num Meth Eng* doi:10.1002/cnm.1001 (2007).

[4] S. Bordas, M. Duflot. “Derivative recovery and a posteriori error estimate for extended finite elements”. *Comp Meth App Mech Eng*, Vol. **196**, pp. 3381-3399, (2007)

[5] M. Duflot, S. Bordas. “A posteriori error estimation for extended finite element by an extended global recovery”. *Int J Num Meth Eng*. Submitted (2007)

[6] J.J. Ródenas, O.A. González-Estrada, J.E. Tarancón, F.J. Fuenmayor. “A recovery-type estimator for the extended finite element method base don *singular+smooth* stress field splitting”, *Int J Num Meth Eng*. Submitted (2007)