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DIRECT METHODS DERIVED FROM LINEAR SOLUTION METHODS WITH SPATIAL VARIATION OF MODULI

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ABSTRACT

There are a collection of direct methods and simplified analysis methods for elastic-plasticity, all of which rely upon iterative processes involving the solution of linear solutions with spatially varying linear moduli. Their interest arises from the ability to implement in standard finite element codes, thereby giving the potential for generally applicable methods available for design use. Hence the main thrust for these developments has been to provide facilities for industries involved in the design of complex metallic structures subjected to severe cycles of loading, sometimes at high temperature.

There are essentially three strands to these developments. Jones [2] and Marriot [4] developed simple lower bound limit load methods, the Reduced Modulus Method, by adapting linear elastic analysis. These methods were extended by Mackenzie and Boyle[3] to include both upper and lower bounds as the Elastic Compensation Method. Seshandri has developed a number of related methods, the R-node method and the GLOSS method [7] that incorporate great insight into structural behaviour. All these methods may be regarded as heuristic methods in the sense that they are imaginative and practical but lack a firm theoretical basis. There is no guarantee of convergence and, as long as the rules of limit analysis and shakedown are adhered to, upper and lower bounds are obtained but they may not be optimal. In addition, as kinematic finite element methods are used, the lower bounds are, at best, only lower bounds to the optimal upper bound corresponding to the finite element mesh.

The second strand that follows from this work has been the attempts by the Ponter *et al* [1,5,6] to derive strictly convergent direct methods that use the same class of representation, linear solutions with spatially varying linear moduli. This has resulted in a class of methods referred to as Linear Matching Methods and arose originally from an attempt to prove the convergence of the Elastic Compensation Method [5]. For shakedown this produces a strictly convergent upper bound method that converges to

the least upper bound associated with class of displacement fields, e.g. those associated with a finite element mesh[6]. As both compatibility and equilibrium (for finite elements in a Galerkin sense) are satisfied for each iterative solution, both upper and associated lower bounds are produced that converge to a common value, the least upper bound. However the lower bounds are not generally monotonically convergent and no independent convergent lower bound method appears to exist. Extensions to ratchet boundaries [1] and creep have been discussed. Application to Life Assessment methods have been adopted by British Energy in the UK.

The third strand is given by the method of Zarka[8] which is generally not described as a direct method but a simplified analysis method, derived from a particular property of a linear kinematically hardening model. By reducing the hardening modulus Zarka observes that perfectly plastic bounding solutions may be derived.

All these three strands have represented independently developed parallel streams with similar and overlapping methods and objectives. The paper describes a systematic comparison of all these methods with the objective of finding a general theoretic framework with which such methods may be discussed. Emphasis is placed on the circumstances when sufficient conditions for convergence can be found.

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