8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

IMPLICIT GRADIENT PLASTICITY MODELLING OF SIZE EFFECTS

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Key Words: *Size effects, Gradient plasticity, Computational Plasticity.*

ABSTRACT

Experiments on thin metallic specimens may exhibit trends which cannot be captured by classical plasticity theories. Compared with predictions based on classical plasticity, using the bulk properties of the material, a strengthening effect is often observed when the thickness of specimens is diminished to the order of a few microns, see e.g. [1]. A range of enriched plasticity theories have been proposed in the literature which aim to improve predictions at such spatial scales. Perhaps the best known are the strain-gradient plasticity theory proposed by Fleck & Hutchinson [2] and the family of gradient plasticity theories developed by Aifantis and co-workers, e.g. [3] – a generalisation of the latter has been proposed by Fleck & Hutchinson in [4]. Both classes introduce a length scale – on the order of microns – below which gradients of (plastic) deformation significantly contribute to the predicted yield strength and/or hardening response. They also require non-standard boundary conditions which give rise to boundary layers in which the plastic flow is restricted (or sometimes promoted). Both effects result in a strengthening with decreasing size which generally agrees well with experimental data.

A disadvantage of the two above-mentioned frameworks is that their structure is rather unfavourable in terms of computational implementation. For the former Fleck–Hutchinson theory [2] this is due to the fact that a higher-order equilibrium equation must be solved, which features second-order gradients of the displacement field. This imposes continuity requirements on the displacement field which cannot easily be met by finite element discretisations. The more recent Fleck–Hutchinson theory [4], as well as the underlying theories due to Aifantis and co-workers, feature a partial differential equation in terms of the effective plastic strain which is valid only in that part of the domain which is deforming plastically. As a consequence, the non-standard boundary conditions must be applied at the internal boundary between the elastic and the plastic region. The fact that this boundary will generally evolve during an analysis significantly complicates the finite element implementation of the theory, particularly if implicit time integration is to be used.

In the present contribution we reformulate the simplest version of the more recent Fleck–Hutchinson theory [4], which basically coincides with the earlier proposal by Aifantis [3], such that the non-standard part of the boundary value problem is always defined on the entire (elastic plus plastic) problem domain.

Starting from an assumed form of the free energy potential, a set of governing equations is derived by considering the global dissipation inequality for a deforming body. The resulting equations show some similarity with the implicit gradient plasticity framework proposed by Engelen et al. [5]. In particular, the same modified Helmholtz equation featuring in this framework is retrieved from the thermodynamics. Associated with this equation is a boundary contribution which – like in the Fleck–Hutchinson theory – can be interpreted as a higher-order traction. The solution of the Helmholtz equation enters the yield stress in an additional hardening contribution, which is more pronounced in the presence of strong plastic strain gradients.

The finite element implementation of the implicit gradient formulation largely follows that of the coupled damage–plasticity framework of Reference [5]. A two-field discretisation is employed, using quadratic shape functions for the displacements and bilinear shape functions for the nonlocal effective plastic strain. A monolithic, full Netwon–Raphson algorithm allows one to solve the resulting algebraic equations in an efficient manner. The ability of the framework to capture size effects is demonstrated by simulations of the bending experiments of Stölken & Evans [1]. Moment–curvature responses predicted by these simulations show good agreement with an analytical solution as well as with the Fleck–Hutchinson theory and experiments.

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