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A DISCONTINUOUS GALERKIN FORMULATION FOR GRADIENT PLASTICITY AT FINITE DEFORMATIONS

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ABSTRACT

Gradient plasticity formulations provide a framework for the description of scale dependent phenomena and render mesh independent descriptions of softening media. As such, gradient plasticity formulations have been widely utilised to describe material failure processes. The non–standard higher–order contributions arising in gradient plasticity formulations renders the framework of the classical finite element method inappropriate. The discontinuous Galerkin finite element method allows these higher–order contributions to be treated in an elegant and effective manner. The model of gradient plasticity considered here extends the classical von Mises yield criterion via the addition of the Laplacian of the isotropic hardening parameter [1,2]. A discontinuous Galerkin framework for this model of gradient plasticity restricted to the infinitesimal strain regime was detailed by the authors in [3,4]. In this work we consider the extension of the gradient plasticity model to the finite deformation regime. The form of the finite strain plasticity model adopted, originally proposed by Simo [5], preserves the essential ingredients of the return mapping algorithms of the infinitesimal theory.

The plastic flow law based upon the von Mises yield criterion imposes isochoric plastic deformation thereby rendering the solution of the problem using conventional lower–order elements susceptible to locking. Locking related problems can be avoided by utilising higher–order finite elements but these in turn increase the computational expense. In addition, the discontinuous Galerkin formulation is most effectively implemented using lower–order finite elements as the edges of elements remain straight and the absence of mid–nodes decreases the computational expense. The method of enhanced strains for geometrically non-linear problems, pioneered by Simo & Armero [6,7], is utilised to provide a locking free response for lower–order elements.

This work focuses on algorithmic and computational aspects of the problem. In particular, it is shown that the predictor–corrector algorithms of classical plasticity are readily extended to the case of gradient plasticity, and to discontinuous Galerkin formulations. The form of the consistent tangent modulus is established for the case of gradient plasticity. A selection of numerical examples is presented and discussed with a view to illustrating aspects of the approximation scheme and algorithms, as well as features of the model of gradient plasticity adopted here.

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