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h-Multigrid for Higher Order Space-Time Discontinuous Galerkin Discretizations of the Compressible Navier-Stokes Equations

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ABSTRACT

Discontinuous Galerkin (DG) methods have grown very popular in recent years due to the relative ease with which the mesh and polynomial order of the basis functions can be (locally) adapted. As opposed to continuous finite element methods, DG methods do not require global continuity of the finite element approximation, making such local *hp*-refinement less difficult. Although originally developed for hyperbolic equations, DG methods have recently also been extended to (incompletely) parabolic equations, in particular the compressible Navier-Stokes equations. The feasibility of using DG discretizations for the spatial discretization of the compressible Navier-Stokes equations has been demonstrated by Bassi and Rebay [1, 2] and Baumann and Oden [3].

In Klaij et. al. [5] we extended the DG method to a space-time discretization of the compressible Navier-Stokes equations. This algorithm uses linear discontinuous basis functions, both in space and time, and results in a second order accurate numerical discretization. It is an extension of the space-time DG method developed by van der Vegt and van der Ven [8, 9] for inviscid compressible flows. The space-time DG method is particularly well suited for problems requiring moving and deforming meshes, such as occur in fluid-structure interaction. The main benefit of a space-time DG method is that it does not require data interpolation and automatically results in a conservative scheme on moving and deforming meshes.

In this presentation we will discuss the extension of a second-order space-time discretization for the compressible Navier-Stokes equations to higher order accuracy. Apart from using higher order polynomial basis functions, a key issue will be the efficient solution of the nonlinear algebraic equations resulting from the space-time discretization. For this purpose we extend the coupled pseudo-time multigrid technique, developed in Klaij et al. [4, 6], to higher order discretizations. This pseudo-time multigrid algorithm preserves the locality of the DG discretization, which is beneficial for parallel computing and does not involve a global Jacobian matrix as Newton methods.

The extension to higher order DG discretizations is challenging because the development of suitable relaxation schemes is no longer straightforward [7]. As relaxation schemes for the

multigrid algorithm, we use explicit multi-stage iterations that may be interpreted as special Runge-Kutta methods. The stage parameters can be optimized to ensure good multigrid smoothing properties.

In order to predict the multigrid behavior, we make use of classical local mode Fourier analysis for a model problem: the scalar advection-diffusion equation. For arbitrary Courant and cell Reynolds numbers, we compute multigrid convergence rates and we find that explicit Runge-Kutta smoothing is suitable for solving the time-dependent advection-diffusion equation. This motivates us to also apply the h-multigrid method to higher order space-time DG discretizations of the compressible Navier-Stokes equations. In this presentation we will also discuss numerical experiments for laminar, steady and unsteady flows that demonstrate that the computational effort is significantly reduced by using h-multigrid.

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