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THREE LEVEL FINITE ELEMENT METHOD

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ABSTRACT

Many important problems in solid mechanics have complex material structure. Applications run from large scale to the nano scale. For example, this could be an earth dam built from large concrete blocks or a film covered by nano particles. Today experimental mechanics is able to give quite detailed material description. To infer from it the mechanical behavior we need very fine discretization which often leads to prohibitive expensive computation. To solve the problem several approaches were proposed and are used, homogenization, multiscale computation and many others. We present here a new method based on the finite element computation which is an extension of the two scale finite element method, [1].

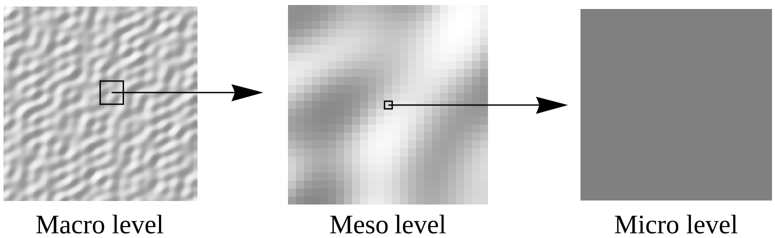

Figure 1: Three level discretization.

The method is described within the context of a linear plane stress problem although it works equally fine for three dimensional linear problems and even for nonlinear problems where the method could be used at each iteration step, [1]. Figure 1 shows three discretization levels. The macro level has a complex material structure and coincides with the size of the structure. At the micro level the material is homogeneous. In a standard finite element method the micro level is discretized into finite elements and their element stiffness matrices are assembled into the global stiffness matrix. This results in a very large matrix. In a two scale finite element method interior micro degrees of freedom are statically condensed to the boundary degrees of freedom. Thus the macro problem has for the degrees of freedom only the micro boundary degrees of freedom and this significantly reduces the size of the macro global matrix. In the three level finite element method an intermediate level is introduced. After micro level condensation another condensation is performed. This time the interior meso level degrees of freedom are condensed to the boundary meso level degrees of freedom. As a result, the size of the macro global matrix is further reduced.

Efficiency η of the three level FEM is estimated by the ratio of the number of the floating point operations in solving the linear systems of the standard and the three level FEM. Efficiency and the corresponding optimal numbers of meso and micro cells are problem dependent. For example we consider a scalar problem where each micro cell has its own material properties. Let us denote their total number by p^2 and by M^2 and m^2 the number of meso and micro cells. Then $p = Mm$. It turns out that for large p the efficiency is of order $\eta = O(p^{2/3})$ while the optimal number of meso cells is $M = (1 - 1/2\mu)^{2/3}(6 - 1/\mu)p^{1/3}$. Here μ^2 is the number quadrilateral bilinear elements of the FEM discretization of each micro cell. On the other hand, if the problem is homogeneous and we just like to use a three level FEM, then the mesh size $h = \frac{1}{Mm\mu}$ is given. For small h the efficiency is of order $\eta = O(h^{-6/7})$ and the optimal values are $M = \beta h^{-1/7}$ and $m = \beta h^{-2/7}$, where $\beta = (90)^{1/7}$. For large p or small h the efficiency of the three level FEM is thus very high. One can envisage even multilevel FEM. However, as higher levels require more static condensations it turns out that on a single processor computer the optimal number of levels is three.

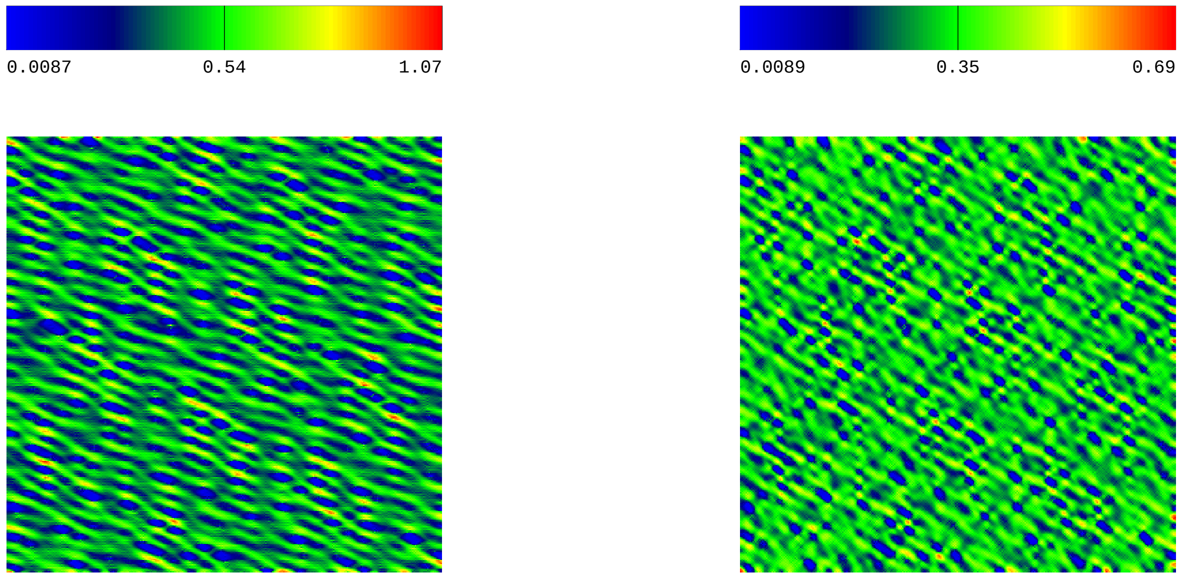

Figure 2: Plot of the maximal normal and shear stresses for the uniaxial and shear deformation.

For a numerical example we present in Figure 2 distribution of the maximal normal and shear stresses. The domain is a square with material distribution shown in Figure 1 where Young modulus is proportional to the gray level in the picture and varies from 1 GPa (black color) to 100 GPa (white color). Such picture can be obtained for example by the atomic force acoustic microscopy. A bitmap picture of the structure has 17 different gray levels and 320 times 320 pixels. Hence $p = 320$. Each micro element is discretized into 4 quadratic quadrilateral elements. The total number of degrees is $6\ 553\ 000$. Using a three level FEM the problem can be solved using a laptop. This promises that using three level FEM 3D problems with complex structure could be readily solved on a mainframe computer.

REFERENCES

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