Scattering of a sound wave on a vortex in Bose-Einstein condensates

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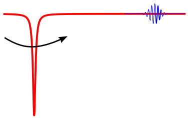
RPMBT14

Outline

Classical fluids

- What happens if a plane wave impacts a vortex?



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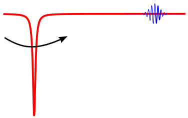

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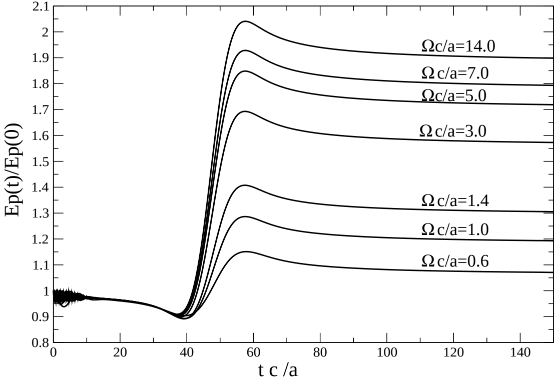
- It is possible to extract some of the vortex rotational energy.



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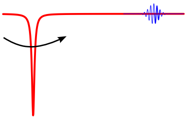

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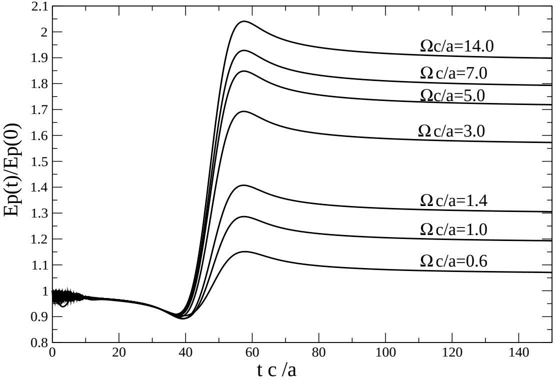

- The phenomenon is called Superradiance

Classical $\longrightarrow$ Quantum

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- In particular, one usually makes the analogy between a rotating black hole and a vortex in a BEC (e.g. cosmolab)

- However. . . the analogy is valid only at the perturbative level!!

What happens to a confined BEC?

Question: Is it possible to transition from a vortex to a state without vorticity as a result of the scattering of a sound wave?

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- The BEC has its vorticity quantized.
- In current experiments, BECs are spatially confined.

The Gross-Pitaevskii energy

The steady states of a BEC at $T = 0$, ψ, are minima of the functional

$$E[\psi] = \int d^3r \left[\frac{\hbar^2}{2m} |\nabla\psi|^2 + V_{ext}(\mathbf{r})|\psi|^2 + \frac{g}{2} |\psi|^4 \right]$$

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Some facts:

- Vortices are steady states of the functional.
- They are not ground states (unless the system is set into rotation).

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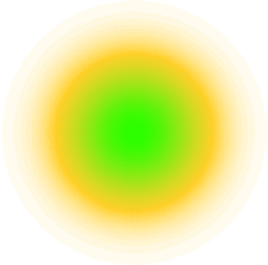

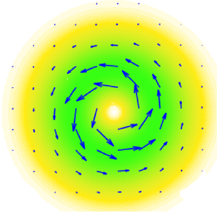

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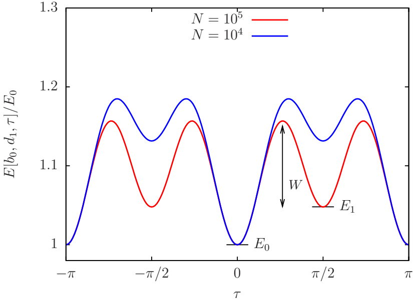
- Populations: $N_0 = N\cos^2\tau$ and $N_1 = N\sin^2\tau$
- $\langle L_z\rangle = \hbar N_1/N$

PRO: The energy can be written analytically. Extremizing, we found the two states ψ_0 and ψ_1.

Energy barrier



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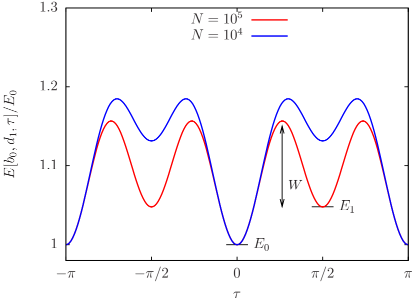

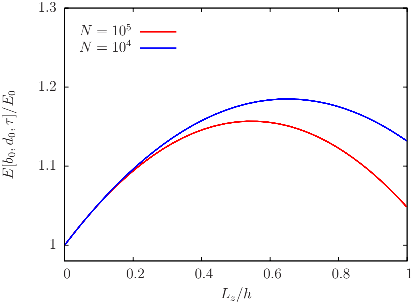

For large interactions, the model predicts $W/\mathrm{E}_0 \simeq 10\%$. Therefore,

- We expect that it will be possible to overcome the energy barrier by supplying enough energy.

Variational dynamics

Starting from the Lagrangian of the system

$$\mathcal{L}[\psi, t] = \int \left[\frac{i\hbar}{2} \left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) - \frac{\hbar^2}{2m} |\nabla \psi|^2 - V_{ext}(\mathbf{r}) |\psi|^2 - \frac{g}{2} |\psi|^4 \right] d\mathbf{r}$$

and a parametrization of the wavefunction $\psi = \psi(\{\alpha\})$, we numerically solve the Euler-Lagrange equations for the α's

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} \right) = \frac{\partial \mathcal{L}}{\partial \alpha}.$$

Our parametrization

- $\psi(\mathbf{r}) = a(t)\psi_0(\mathbf{r}) + b(t)\psi_1(\mathbf{r}), \qquad \text{with } a(t), b(t) \in \mathbb{C}.$

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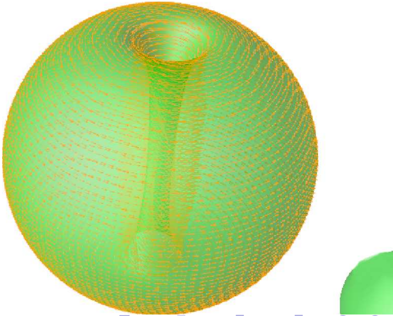
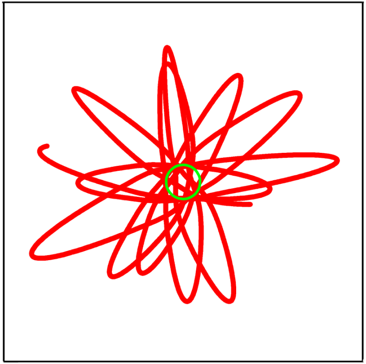
Initial state

$a(0) = 0$, $b(0) \neq 0$, $c(0) \neq 0$, $\mathbf{k}(0) = -k_0\,\hat{x}$ and $\mathbf{r}_0(0)$ outside the condensate.



Wavepacket orbits

For ^{87}Rb parameters with $\mathrm{N} = 10^5$ and $\mathrm{N_p/N} \simeq 8\%$.



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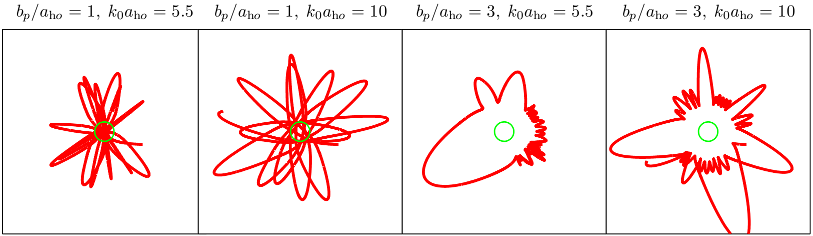
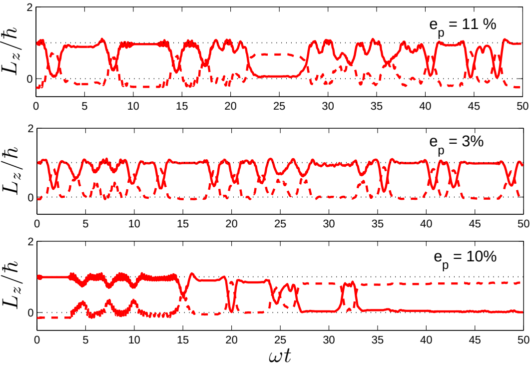


- The bigger b_p, the lower the density and less penetrating wavepacket.
- Changes in the L_z manifest themselves through, e.g., changes in the orbits area.

Angular Momentum



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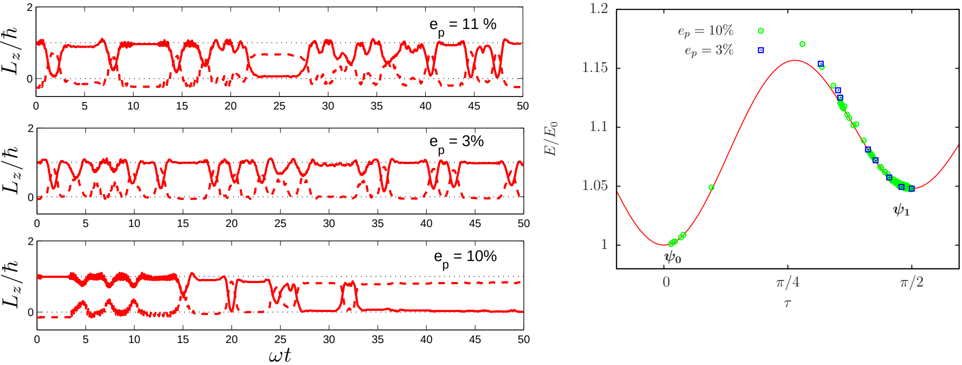
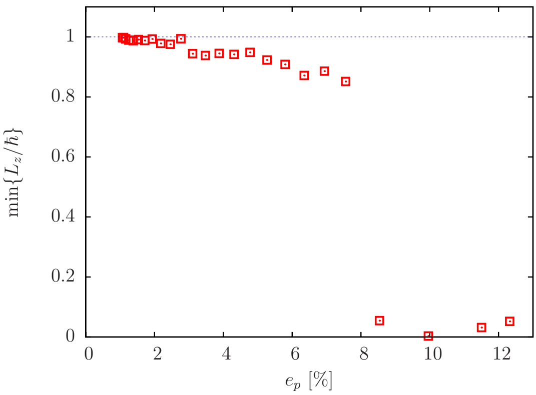


- The wavepacket energy is efficiently transferred to the system.

Minimum of L_z



Summary and outlook

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Future directions

- Numerical solution of the GPE
  - diffusion of the condensate and wavepacket.
  - excitation of collective modes.
- What if the vortex and the wavepacket does not belong to the same condensate?
- Two-dimensional geometry.