

# Fermi Hypernetted chain description of doubly closed shell nuclei

F. Arias de Saavedra,

Departamento de Física Atómica, Molecular y Nuclear,  
Universidad de Granada, E-18071 Granada, Spain

C. Bisconti and G. Co',

Dipartimento di Fisica, Università del Salento,  
and INFN sez. di Lecce, I-73100 Lecce, Italy

*Phys. Rev.* **C73** (2006), 054304; **C75** (2007) 054302;

*Phys. Rep.* in press (arXiv:0706.3792)

# Outline

- Introduction.

# Outline

- Introduction.
- Ground-state energies.

# Outline

- Introduction.
- Ground-state energies.
- One- and two-body distribution functions.

# Outline

- Introduction.
- Ground-state energies.
- One- and two-body distribution functions.
- Momentum distributions.

# Outline

- Introduction.
- Ground-state energies.
- One- and two-body distribution functions.
- Momentum distributions.
- Natural orbits.

# Outline

- Introduction.
- Ground-state energies.
- One- and two-body distribution functions.
- Momentum distributions.
- Natural orbits.
- Quasi-hole wave functions and spectroscopic factors.

# Outline

- Introduction.
- Ground-state energies.
- One- and two-body distribution functions.
- Momentum distributions.
- Natural orbits.
- Quasi-hole wave functions and spectroscopic factors.
- Conclusions.

# Introduction

$$H = - \sum_{i=1}^A \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{j>i=1}^A V_{ij} + \sum_{k>j>i=1}^A V_{ijk} .$$

# Introduction

$$H = - \sum_{i=1}^A \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{j>i=1}^A V_{ij} + \sum_{k>j>i=1}^A V_{ijk} .$$

$$V_{ij} = \sum_{p=1}^8 v^p(r_{ij}) O_{ij}^p$$

# Introduction

$$H = - \sum_{i=1}^A \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{j>i=1}^A V_{ij} + \sum_{k>j>i=1}^A V_{ijk} .$$

$$V_{ij} = \sum_{p=1}^8 v^p(r_{ij}) O_{ij}^p$$

$$O_{ij}^{p=1,8} = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, (\mathbf{L} \cdot \mathbf{S})_{ij}] \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j]$$

Interaction used: A8'+UIX

# Variational principle

$$\delta E[\Psi] = \delta \frac{<\Psi|H|\Psi>}{<\Psi|\Psi>} = 0.$$

# Variational principle

$$\delta E[\Psi] = \delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0.$$

Trial wave function

$$\Psi(1, \dots, A) = \mathcal{F}(1, \dots, A) \Phi(1, \dots, A)$$

$\Phi$  Slater det. with Woods-Saxon pot.

# Variational principle

$$\delta E[\Psi] = \delta \frac{<\Psi|H|\Psi>}{<\Psi|\Psi>} = 0.$$

Trial wave function

$$\Psi(1, \dots, A) = \mathcal{F}(1, \dots, A)\Phi(1, \dots, A)$$

$\Phi$  Slater det. with Woods-Saxon pot.

$$\mathcal{F}(1, \dots, A) = \mathcal{S} \left( \prod_{j>i=1}^A \sum_{p=1}^6 f_p(r_{ij}) O_{ij}^p \right)$$

# Variational principle

$$\delta E[\Psi] = \delta \frac{<\Psi|H|\Psi>}{<\Psi|\Psi>} = 0.$$

Trial wave function

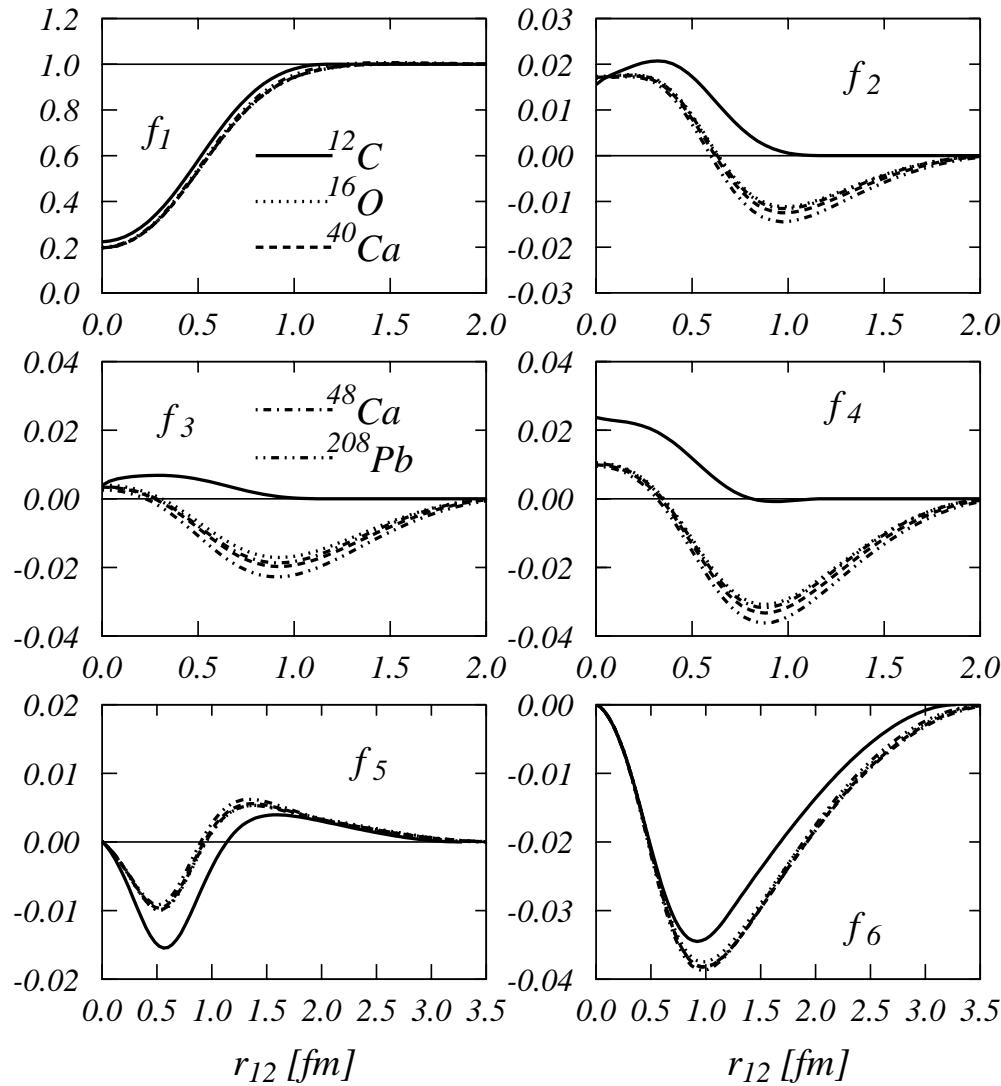
$$\Psi(1, \dots, A) = \mathcal{F}(1, \dots, A) \Phi(1, \dots, A)$$

$\Phi$  Slater det. with Woods-Saxon pot.

$$\mathcal{F}(1, \dots, A) = \left( \prod_{j>i=1}^A f_1(r_{ij}) \right) \mathcal{S} \left( \prod_{j>i=1}^A \left( 1 + \sum_{p=2}^6 \frac{f_p(r_{ij})}{f_1(r_{ij})} O_{ij}^p \right) \right)$$

$f_p$  obtained with Euler procedure.

# Euler correlations



# Ground-state energies

AV8'+UIX	$^{12}\text{C}$	$^{16}\text{O}$	$^{40}\text{Ca}$	$^{48}\text{Ca}$	$^{208}\text{Pb}$
$T$	27.13	32.33	41.06	39.64	39.56
$V_{2-body}$	-29.38	-38.63	-49.36	-46.95	-48.88
$V_{Coul}$	0.67	0.86	1.97	1.57	3.97
$T + V_2$	<b>-1.58</b>	<b>-5.34</b>	<b>-6.34</b>	<b>-5.74</b>	<b>-5.35</b>
$V_{3-body}$	0.67	0.86	1.76	1.61	1.91
$E$	<b>-0.91</b>	<b>-4.48</b>	<b>-4.58</b>	<b>-4.14</b>	<b>-3.43</b>
$E_{exp}$	<b>-7.68</b>	<b>-7.97</b>	<b>-8.55</b>	<b>-8.66</b>	<b>-7.86</b>

Estimation of the rest of the channels in the interaction using nuclear matter calculations.

	$^{12}\text{C}$	$^{16}\text{O}$	$^{40}\text{Ca}$	$^{48}\text{Ca}$	$^{208}\text{Pb}$
$\langle k_F \rangle$	1.09	1.09	1.19	1.19	1.21
$E$	-0.91	-4.48	-4.58	-4.14	-3.43
$\Delta E$	-0.93	-0.93	-1.37	-1.37	-1.48
$E + \Delta E$	-1.84	-5.41	-5.95	-5.51	-4.91

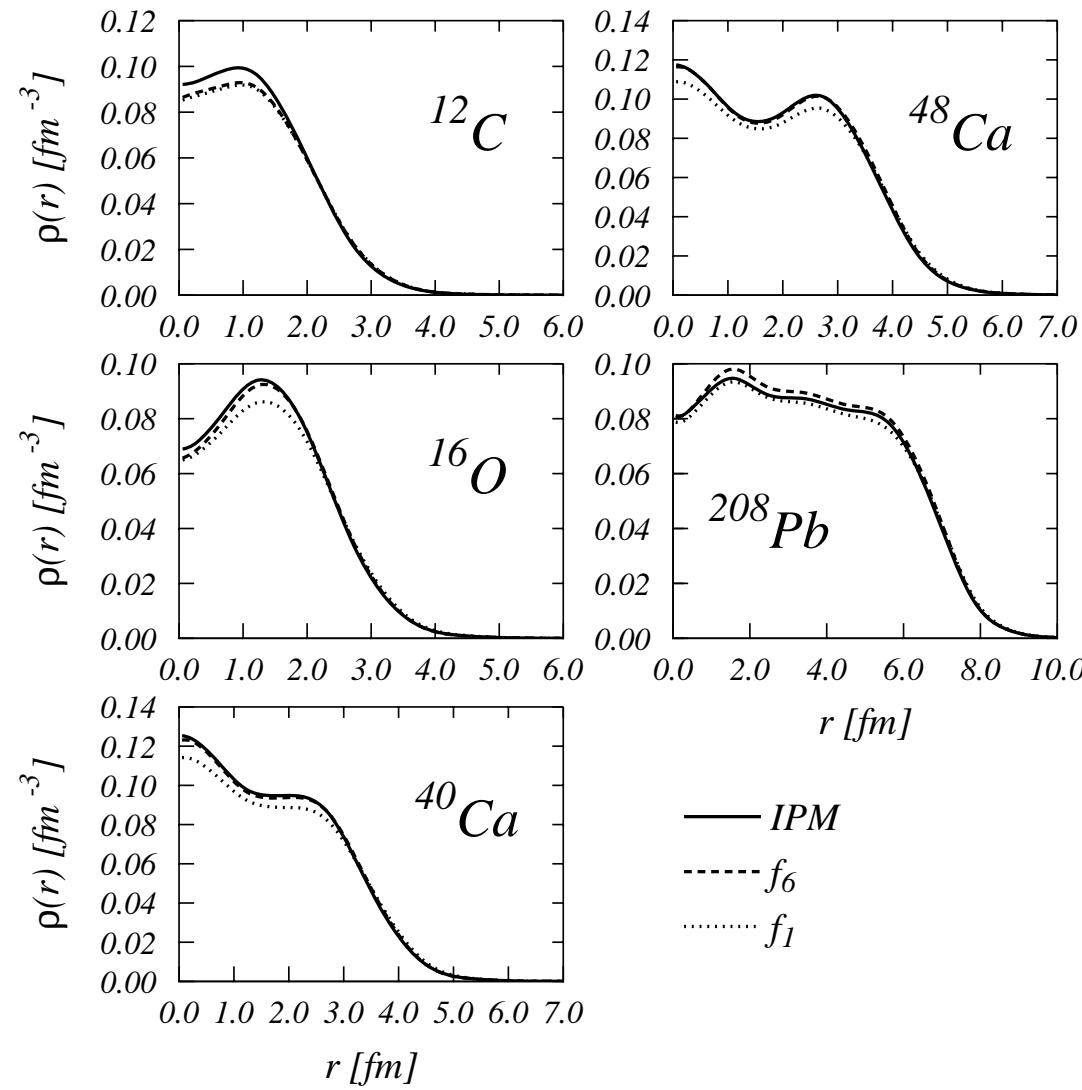
# OBDF and TBDF

$$\rho^{t_1}(\mathbf{r}_1) = \frac{\mathcal{N}_{t_1}}{\langle \Psi | \Psi \rangle} \int dx_2 \dots dx_A \times \Psi^*(x_1, \dots, x_A) P_1^{t_1} \Psi(x_1, \dots, x_A) ,$$

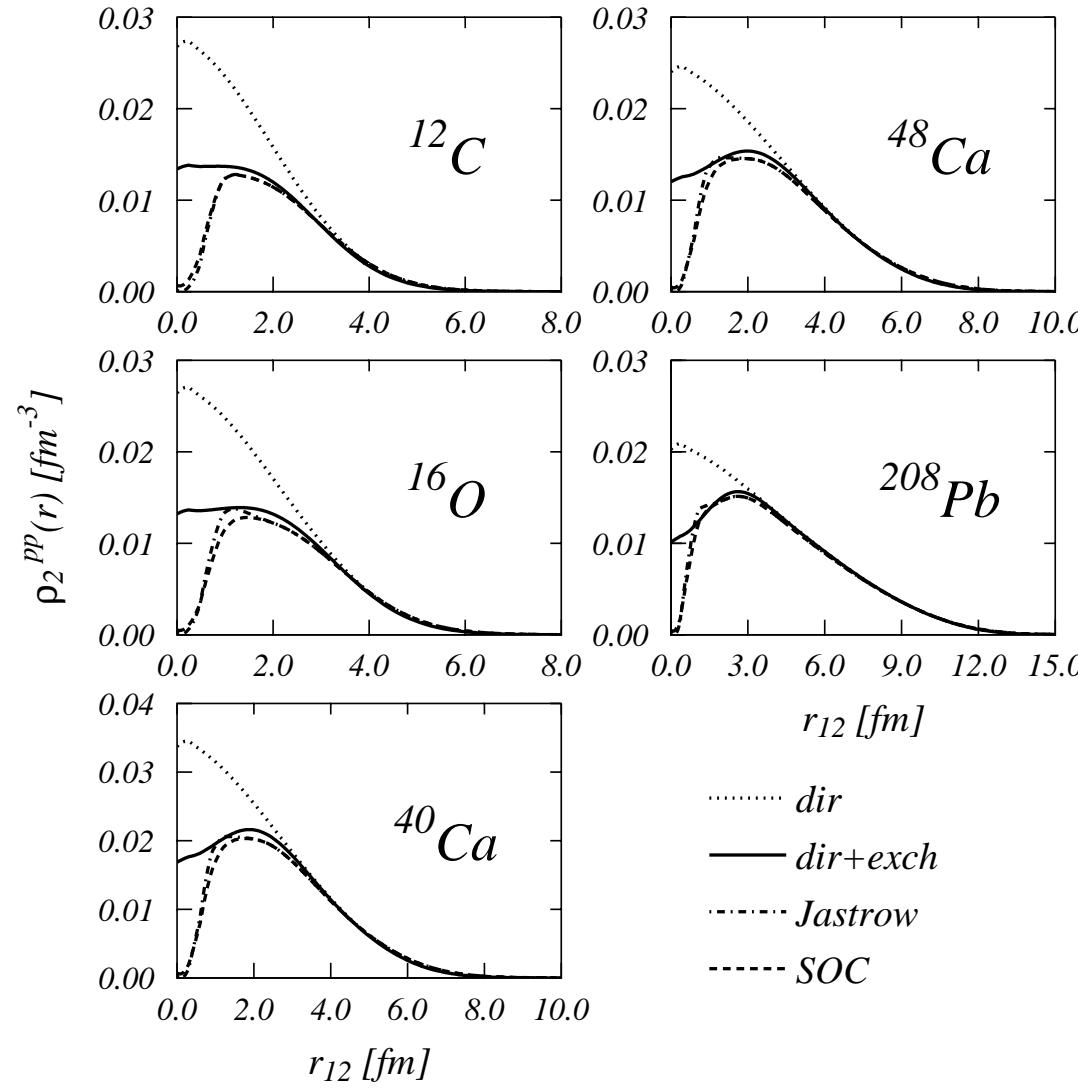
$$\rho_2^{q,t_1t_2}(\mathbf{r}_1, \mathbf{r}_2) = \frac{\mathcal{N}_{t_1}(\mathcal{N}_{t_2} - \delta_{t_1t_2})}{\langle \Psi | \Psi \rangle} \int dx_3 \dots dx_A \times \Psi^*(x_1, \dots, x_A) P_1^{t_1} O_{12}^q P_2^{t_2} \Psi(x_1, \dots, x_A)$$

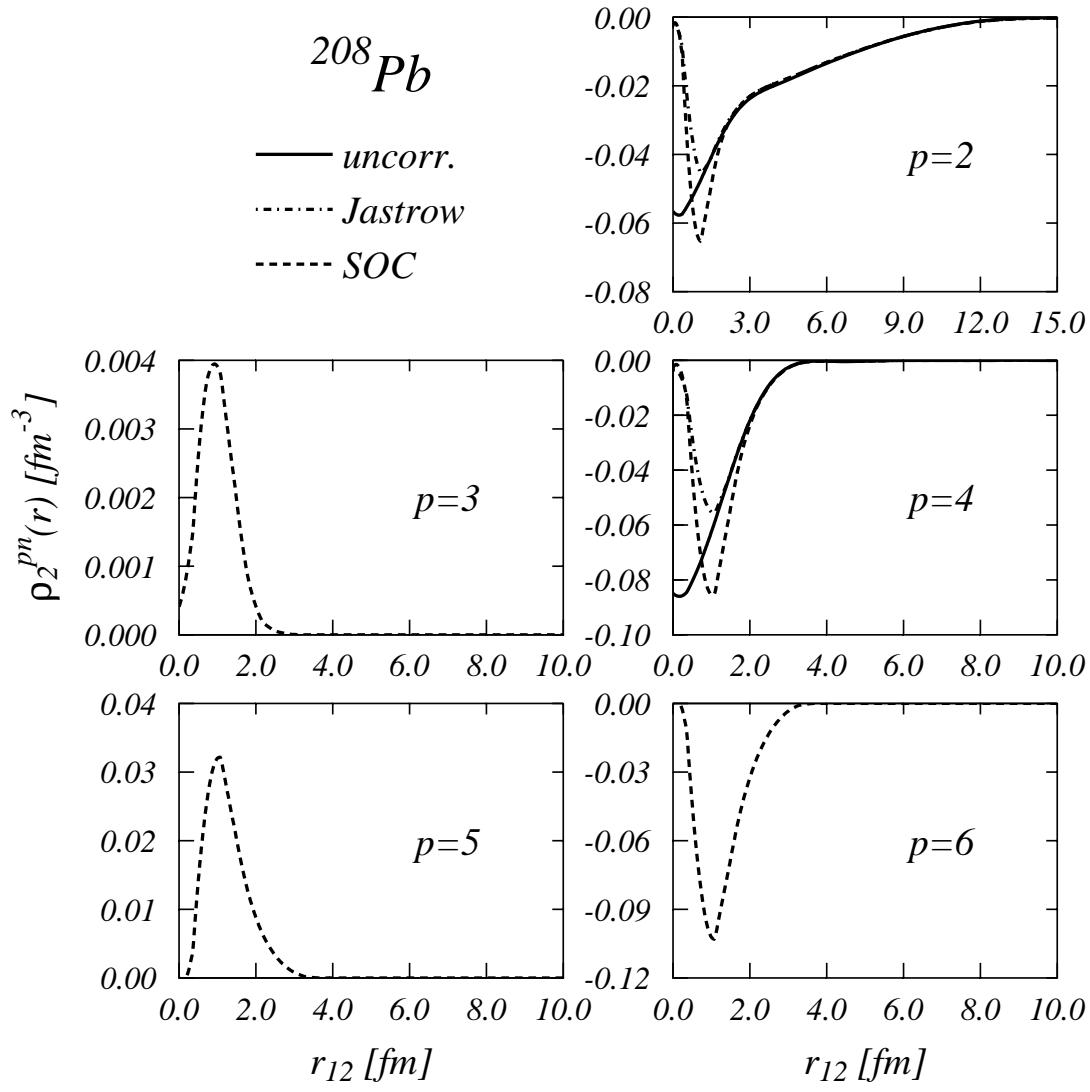
$$\mathcal{N}_p = Z \text{ and } \mathcal{N}_n = N$$

# Neutron density functions



$$\rho_2^{1,pp}(r_{12})$$



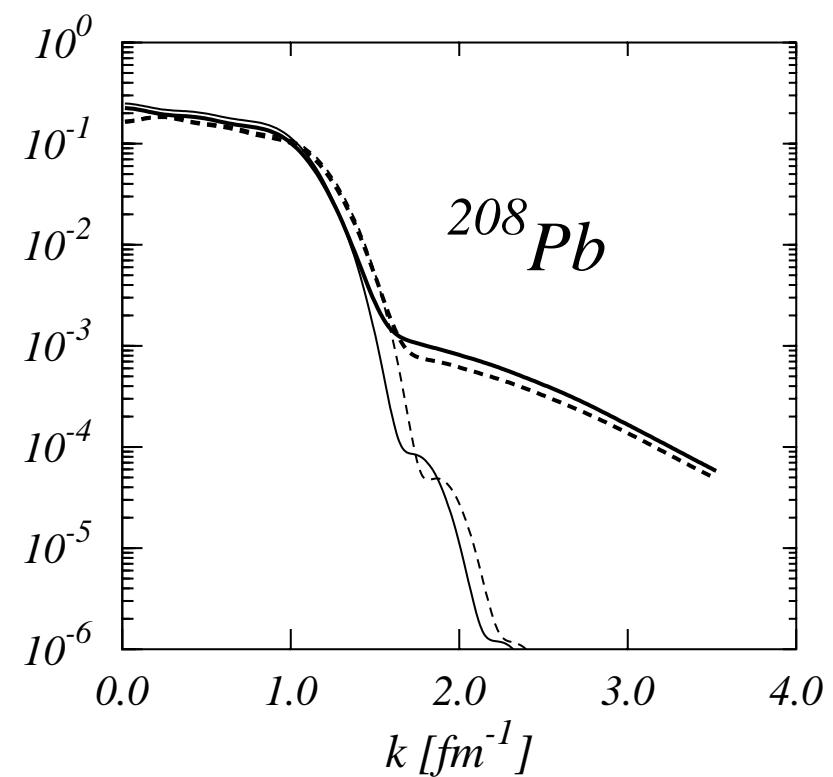
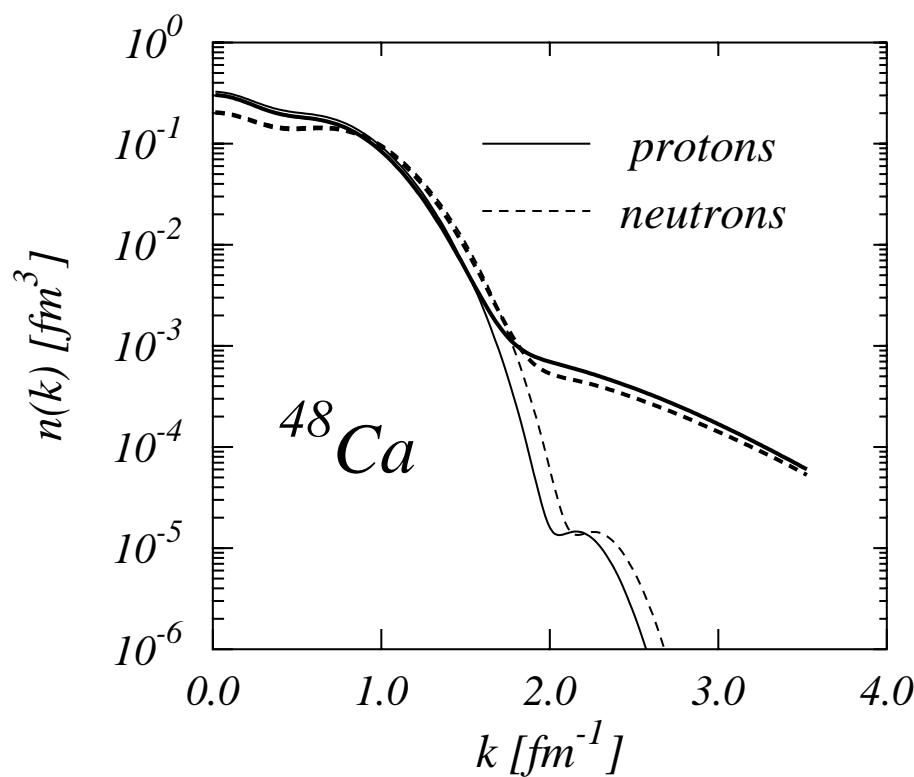


# Momentum distribution

$$\begin{aligned}\rho(x_1, x'_1) &= \sum_{s,s',t} \rho^{s,s';t}(\mathbf{r}_1, \mathbf{r}'_1) \chi_s^+(1) \chi_t^+(1) \chi_{s'}^+(1') \chi_t(1') \\ &= \frac{A}{\langle \Psi | \Psi \rangle} \int dx_2 \dots dx_A \Psi^\dagger(x_1, \dots, x_A) \Psi(x'_1, \dots, x_A)\end{aligned}$$

$$\rho^t(\mathbf{r}_1, \mathbf{r}'_1) = \sum_{s=\pm 1/2} [\rho^{s,s;t}(\mathbf{r}_1, \mathbf{r}'_1) + \rho^{s,-s;t}(\mathbf{r}_1, \mathbf{r}'_1)] ,$$

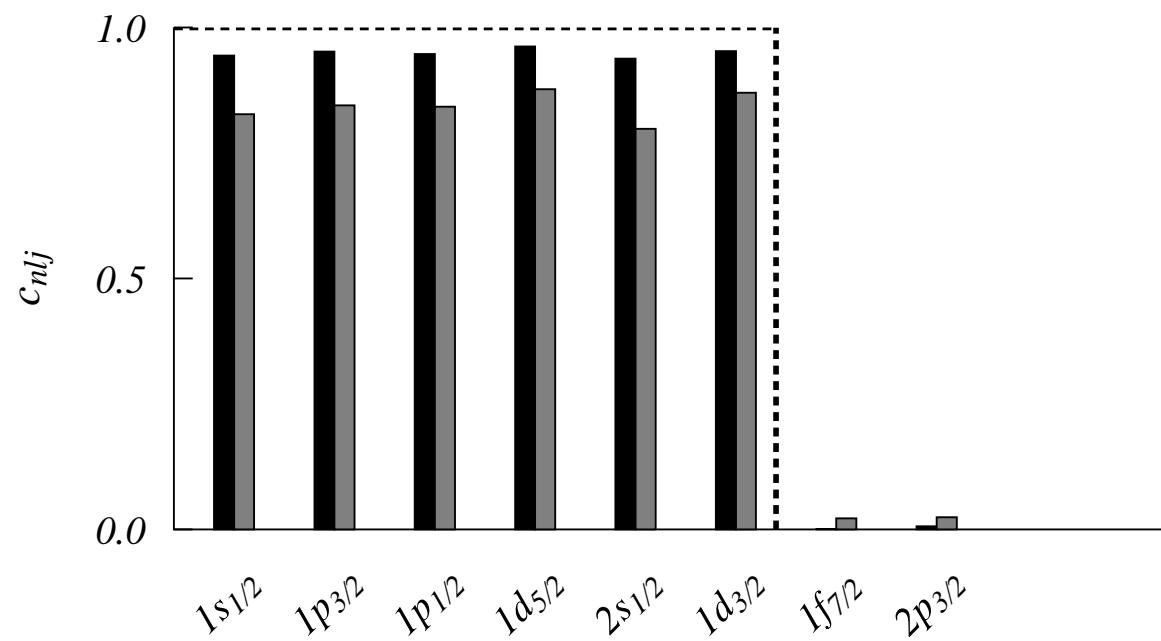
$$n^t(k) = \frac{1}{(2\pi)^3} \frac{1}{\mathcal{N}_t} \int d\mathbf{r}_1 d\mathbf{r}'_1 e^{i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}'_1)} \rho^t(\mathbf{r}_1, \mathbf{r}'_1) ,$$



# Natural orbits

$$\rho^t(\mathbf{r}_1, \mathbf{r}'_1) = \sum_{nlj} c_{nlj}^t \phi_{nlj}^{*, t, NO}(\mathbf{r}_1) \phi_{nlj}^{t, NO}(\mathbf{r}'_1)$$

Occupation numbers of protons for  $^{48}\text{Ca}$ .



# Quasi hole wave functions

$$\psi_{nljm}^t(x) = \frac{\sqrt{A} \langle \Psi_{nljm}^t(1, \dots, A-1) | \delta(x - x_A) P_A^t | \Psi(1, \dots, A) \rangle}{\langle \Psi_{nljm}^t | \Psi_{nljm}^t \rangle^{1/2} \langle \Psi | \Psi \rangle^{1/2}}$$

$$\Psi_{nljm}^t(1, \dots, A-1) = \mathcal{F}(1, \dots, A-1) \Phi_{nljm}^t(1, \dots, A-1)$$

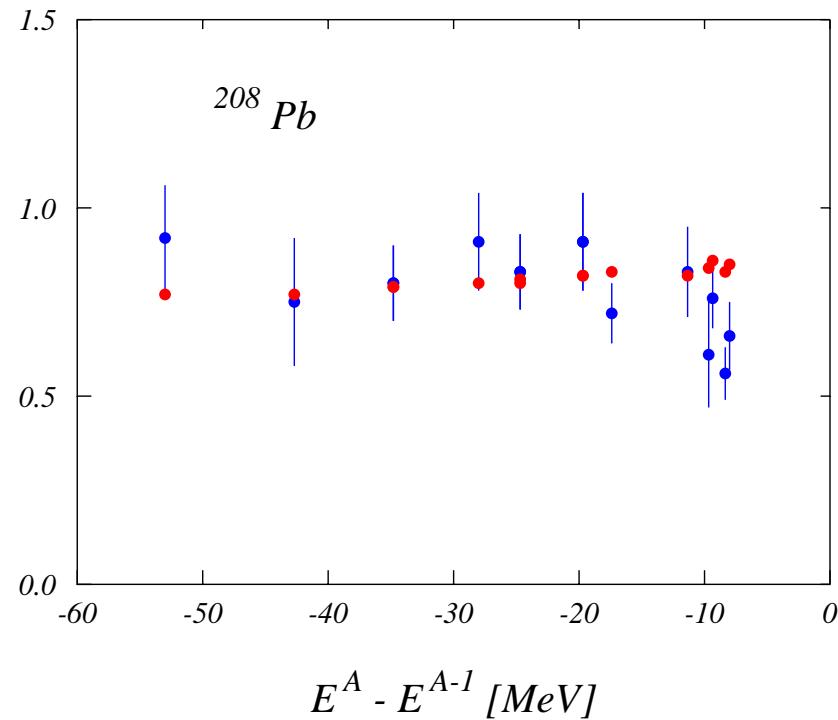
Interested in its radial part:

$$\psi_{nlj}^t(r) = \frac{1}{2j+1} \sum_m \int d\Omega \mathbf{Y}_{lj}^{*m}(\Omega) \psi_{nljm}^t(x)$$

# Spectroscopic factors

$$S_{nlj}^t = \int dr r^2 |\psi_{nlj}^t(r)|^2$$

Proton spectroscopic factors for  $^{208}\text{Pb}$ .



# Conclusions

- CBF calculations in finite nuclei are as accurate as nuclear matter ones.

# Conclusions

- CBF calculations in finite nuclei are as accurate as nuclear matter ones.
- Short-range correlations are important to describe nuclei.