BCS-BEC Cross over: Using the ERG

Niels Walet with Mike Birse, Boris Krippa and Judith McGovern

School of Physics and Astronomy
University of Manchester

RPMBT14, Barcelona, 2007

Outline

Motivation

- Effective field theory has become a much more formal tool.

Motivation

- Effective field theory has become a much more formal tool.
- Very applicable low energy nuclear/nucleon physics

Motivation

- Effective field theory has become a much more formal tool.
- Very applicable low energy nuclear/nucleon physics
- Relies on separation of scales.

Motivation

- Effective field theory has become a much more formal tool.
- Very applicable low energy nuclear/nucleon physics
- Relies on separation of scales.
- Too many similar scales in nuclear matter/strong pairing.

Motivation

- Effective field theory has become a much more formal tool.
- Very applicable low energy nuclear/nucleon physics
- Relies on separation of scales.
- Too many similar scales in nuclear matter/strong pairing.
- Many-body theory of weak repulsive force (“natural”) . [Hammer and Furnstahl, 2000]. Reproduces old results of Ray Bishop.

Motivation

- Effective field theory has become a much more formal tool.
- Very applicable low energy nuclear/nucleon physics
- Relies on separation of scales.
- Too many similar scales in nuclear matter/strong pairing.
- Many-body theory of weak repulsive force (“natural”) . [Hammer and Furnstahl, 2000]. Reproduces old results of Ray Bishop.
- No real theory for strongly interacting systems.

Motivation

- Effective field theory has become a much more formal tool.
- Very applicable low energy nuclear/nucleon physics
- Relies on separation of scales.
- Too many similar scales in nuclear matter/strong pairing.
- Many-body theory of weak repulsive force (“natural”) . [Hammer and Furnstahl, 2000]. Reproduces old results of Ray Bishop.
- No real theory for strongly interacting systems.
- Try to find a simple field-theoretical approach.

Motivation 2

- We have used EFT technique to study “reparametrisation invariance” of many-body techniques

Motivation 2

- We have used EFT technique to study “reparametrisation invariance” of many-body techniques
- We are really interested in the pairing problem.

Motivation 2

- We have used EFT technique to study “reparametrisation invariance” of many-body techniques
- We are really interested in the pairing problem.
- Interested in a very pure problem: pure pairing.

Motivation 2

- We have used EFT technique to study “reparametrisation invariance” of many-body techniques
- We are really interested in the pairing problem.
- Interested in a very pure problem: pure pairing.
- Divergent problem, with old solution: divergence of gap equation is divergence of scattering length. Relate physical quantities, and loose the infinity!

Motivation 2

- We have used EFT technique to study “reparametrisation invariance” of many-body techniques
- We are really interested in the pairing problem.
- Interested in a very pure problem: pure pairing.
- Divergent problem, with old solution: divergence of gap equation is divergence of scattering length. Relate physical quantities, and loose the infinity!
- Work we have done a while ago, but are returning to now.

Motivation 2

- We have used EFT technique to study “reparametrisation invariance” of many-body techniques
- We are really interested in the pairing problem.
- Interested in a very pure problem: pure pairing.
- Divergent problem, with old solution: divergence of gap equation is divergence of scattering length. Relate physical quantities, and loose the infinity!
- Work we have done a while ago, but are returning to now.
- See also Blaizot *et al*, Diehl *et al*.

EFT

- All of low energy nuclear physics is not described by quarks and gluons.

EFT

- All of low energy nuclear physics is not described by quarks and gluons.
- Use effective degrees of freedom: point nucleons and pions (pions are light!)

EFT

- All of low energy nuclear physics is not described by quarks and gluons.
- Use effective degrees of freedom: point nucleons and pions (pions are light!)
- Try to do a perturbative expansion in p/Λ: powercounting.

EFT

- All of low energy nuclear physics is not described by quarks and gluons.
- Use effective degrees of freedom: point nucleons and pions (pions are light!)
- Try to do a perturbative expansion in p/Λ: powercounting.
- p small momentum or pion mass, Λ typical scale (300 MeV)

EFT

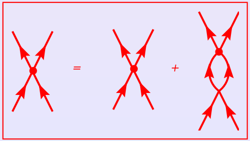
- All of low energy nuclear physics is not described by quarks and gluons.
- Use effective degrees of freedom: point nucleons and pions (pions are light!)
- Try to do a perturbative expansion in p/Λ: powercounting.
- p small momentum or pion mass, Λ typical scale (300 MeV)
- Wilsonian RG for scattering has two fixed points

EFT

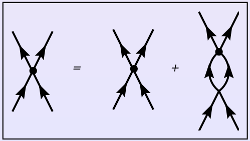
- All of low energy nuclear physics is not described by quarks and gluons.
- Use effective degrees of freedom: point nucleons and pions (pions are light!)
- Try to do a perturbative expansion in p/Λ: powercounting.
- p small momentum or pion mass, Λ typical scale (300 MeV)
- Wilsonian RG for scattering has two fixed points
- Trivial: Weinberg counting

EFT

- All of low energy nuclear physics is not described by quarks and gluons.
- Use effective degrees of freedom: point nucleons and pions (pions are light!)
- Try to do a perturbative expansion in p/Λ: powercounting.
- p small momentum or pion mass, Λ typical scale (300 MeV)
- Wilsonian RG for scattering has two fixed points
- Trivial: Weinberg counting
- Nontrivial: Kaplan-Savage-Wise counting: ladder sum in zeroth order



EFT

- All of low energy nuclear physics is not described by quarks and gluons.
- Use effective degrees of freedom: point nucleons and pions (pions are light!)
- Try to do a perturbative expansion in p/Λ: powercounting.
- p small momentum or pion mass, Λ typical scale (300 MeV)
- Wilsonian RG for scattering has two fixed points
- Trivial: Weinberg counting
- Nontrivial: Kaplan-Savage-Wise counting: ladder sum in zeroth order 

- Reproduces effective range expansion

Few Body Systems

3 body

- See papers by Barfield and Birse

Few Body Systems

3 body

- See papers by Barfield and Birse
- Idea is that we can only look at shallow bound states ($1/r^2$ attraction)

Few Body Systems

3 body

- See papers by Barfield and Birse
- Idea is that we can only look at shallow bound states ($1/r^2$ attraction)
- Consistent power counting: Efimov effect

Few Body Systems

3 body

- See papers by Barfield and Birse
- Idea is that we can only look at shallow bound states ($1/r^2$ attraction)
- Consistent power counting: Efimov effect
- non-trivial RG flow (limit cycle)

Few Body Systems

3 body

- See papers by Barfield and Birse
- Idea is that we can only look at shallow bound states ($1/r^2$ attraction)
- Consistent power counting: Efimov effect
- non-trivial RG flow (limit cycle)

Few Body Systems

3 body

- See papers by Barfield and Birse
- Idea is that we can only look at shallow bound states ($1/r^2$ attraction)
- Consistent power counting: Efimov effect
- non-trivial RG flow (limit cycle)

4 body

Is there much more to learn?

Few Body Systems

3 body

- See papers by Barfield and Birse
- Idea is that we can only look at shallow bound states ($1/r^2$ attraction)
- Consistent power counting: Efimov effect
- non-trivial RG flow (limit cycle)

4 body

Is there much more to learn?

Many body

No separation of scales: $p_f a_0$ second scale (and in NP, effective range 3rd scale).

Link to atomic physics

- Look at dilute atomic systems

Link to atomic physics

- Look at dilute atomic systems
- We have an effective theory of point-like atoms.

Link to atomic physics

- Look at dilute atomic systems
- We have an effective theory of point-like atoms.
- Interacting mainly through zero-range S-wave scattering (plus effective range)

Link to atomic physics

- Look at dilute atomic systems
- We have an effective theory of point-like atoms.
- Interacting mainly through zero-range S-wave scattering (plus effective range)
- Tunable scattering length (Feshbach resonance)

Link to atomic physics

- Look at dilute atomic systems
- We have an effective theory of point-like atoms.
- Interacting mainly through zero-range S-wave scattering (plus effective range)
- Tunable scattering length (Feshbach resonance)
- It may thus be good to apply EFT ideas to such systems....

Exact Renormalization Group

Basic Object

- Work with effective action Γ (see Weinberg, QTF II).

Exact Renormalization Group

Basic Object

- Work with effective action Γ (see Weinberg, QTF II).
- Legendre transform of usual $W_J = \ln Z_J$.

Exact Renormalization Group

Basic Object

- Work with effective action Γ (see Weinberg, QTF II).
- Legendre transform of usual $W_J = \ln Z_J$.
- Functional of *classical* field.

Exact Renormalization Group

Basic Object

- Work with effective action Γ (see Weinberg, QTF II).
- Legendre transform of usual $W_J = \ln Z_J$.
- Functional of *classical* field.
- Whereas functional derivatives of W_J give expectation value of field, functional derivatives of Γ give expectation value of (1PI) Green's functions.

Exact Renormalization Group

Basic Object

- Work with effective action Γ (see Weinberg, QTF II).
- Legendre transform of usual $W_J = \ln Z_J$.
- Functional of *classical* field.
- Whereas functional derivatives of W_J give expectation value of field, functional derivatives of Γ give expectation value of (1PI) Green's functions.

Exact Renormalization Group

Basic Object

- Work with effective action Γ (see Weinberg, QTF II).
- Legendre transform of usual $W_J = \ln Z_J$.
- Functional of *classical* field.
- Whereas functional derivatives of W_J give expectation value of field, functional derivatives of Γ give expectation value of (1PI) Green's functions.

Idea

- Introduced by Wetterich [PLB301 (1993) 90].

Exact Renormalization Group

Basic Object

- Work with effective action Γ (see Weinberg, QTF II).
- Legendre transform of usual $W_J = \ln Z_J$.
- Functional of *classical* field.
- Whereas functional derivatives of W_J give expectation value of field, functional derivatives of Γ give expectation value of (1PI) Green's functions.

Idea

- Introduced by Wetterich [PLB301 (1993) 90].
- Reviews see hep-ph/0005122, cond-mat/0309101.

Exact Renormalization Group

Basic Object

- Work with effective action Γ (see Weinberg, QTF II).
- Legendre transform of usual $W_J = \ln Z_J$.
- Functional of *classical* field.
- Whereas functional derivatives of W_J give expectation value of field, functional derivatives of Γ give expectation value of (1PI) Green's functions.

Idea

- Introduced by Wetterich [PLB301 (1993) 90].
- Reviews see hep-ph/0005122, cond-mat/0309101.
- Add artificial running (RG) to problem.

ERG: The basics I

Zero temperature version–normally finite T

Use a single real scalar field ϕ

- $e^{iW[J]} = \int D\phi\, e^{i(S[\phi]+J\cdot\phi-\frac{1}{2}\phi\cdot R\cdot\phi)},$

ERG: The basics I

Zero temperature version–normally finite T

Use a single real scalar field ϕ

- $e^{iW[J]} = \int D\phi\, e^{i(S[\phi]+J\cdot\phi-\frac{1}{2}\phi\cdot R\cdot\phi)},$
- Here $R(k)$ is our regulator function, which suppresses modes with $q < k$.

ERG: The basics I

Zero temperature version–normally finite T

Use a single real scalar field ϕ

- $e^{iW[J]} = \int D\phi\, e^{i(S[\phi]+J\cdot\phi-\frac{1}{2}\phi\cdot R\cdot\phi)}$,
- Here $R(k)$ is our regulator function, which suppresses modes with $q < k$.
- Expectation value of the field: Solve $\dfrac{\delta W}{\delta J} = \langle\phi\rangle \equiv \phi_c$.

ERG: The basics I

Zero temperature version–normally finite T

Use a single real scalar field ϕ

- $e^{iW[J]} = \int D\phi\, e^{i(S[\phi]+J\cdot\phi-\frac{1}{2}\phi\cdot R\cdot\phi)},$
- Here $R(k)$ is our regulator function, which suppresses modes with $q < k$.
- Expectation value of the field: Solve $\dfrac{\delta W}{\delta J} = \langle\phi\rangle \equiv \phi_c.$
- Legendre transformed effective action is defined by $\Gamma[\phi_c] = W[J] - J\cdot\phi_c + \dfrac{1}{2}\phi_c\cdot R\cdot\phi_c.$

ERG: The basics I

Zero temperature version–normally finite T

Use a single real scalar field ϕ

- $e^{iW[J]} = \int D\phi\, e^{i(S[\phi]+J\cdot\phi-\frac{1}{2}\phi\cdot R\cdot\phi)}$,
- Here $R(k)$ is our regulator function, which suppresses modes with $q < k$.
- Expectation value of the field: Solve $\dfrac{\delta W}{\delta J} = \langle\phi\rangle \equiv \phi_c$.
- Legendre transformed effective action is defined by $\Gamma[\phi_c] = W[J] - J\cdot\phi_c + \dfrac{1}{2}\phi_c\cdot R\cdot\phi_c$.
- $\dfrac{\delta}{\delta\phi_c}\Gamma = J = 0$ no sources.

ERG: The basics II

- The evolution of W is given by

$$
\begin{aligned}
ie^{iW}\partial_k W &= -\frac{i}{2}\int D\phi\,(\phi\cdot\partial_k R\cdot\phi)e^{i(S[\phi]+J\cdot\phi-\frac{1}{2}\phi\cdot R\cdot\phi)},\\
&= -\frac{i}{2}\left(-i\,\frac{\delta}{\delta J}\right)\cdot\partial_k R\cdot\left(-i\,\frac{\delta}{\delta J}\right)e^{iW}\\
&= -\frac{i}{2}e^{iW}(\phi_c\cdot\partial_k R\cdot\phi_c)-\frac{1}{2}e^{iW}\mathrm{Tr}\left[(\partial_k R)\,\frac{\delta\phi_c}{\delta J}\right].
\end{aligned}
$$

ERG: The basics II

- The evolution of W is given by

$$
\begin{aligned}
ie^{iW}\partial_k W &= -\frac{i}{2}\int D\phi\,(\phi\cdot\partial_k R\cdot\phi)e^{i(S[\phi]+J\cdot\phi-\frac{1}{2}\phi\cdot R\cdot\phi)},\\
&= -\frac{i}{2}\left(-i\frac{\delta}{\delta J}\right)\cdot\partial_k R\cdot\left(-i\frac{\delta}{\delta J}\right)e^{iW}\\
&= -\frac{i}{2}e^{iW}(\phi_c\cdot\partial_k R\cdot\phi_c)-\frac{1}{2}e^{iW}\,\mathrm{Tr}\left[(\partial_k R)\frac{\delta\phi_c}{\delta J}\right].
\end{aligned}
$$

- Hence we get $\partial_k W = -\frac{1}{2}(\phi_c\cdot\partial_k R\cdot\phi_c)+\frac{i}{2}\,\mathrm{Tr}\left[(\partial_k R)\frac{\delta\phi_c}{\delta J}\right].$

ERG: The basics II

- The evolution of W is given by

$$
\begin{aligned}
ie^{iW}\partial_k W &= -\frac{i}{2}\int D\phi\,(\phi\cdot\partial_k R\cdot\phi)e^{i(S[\phi]+J\cdot\phi-\frac{1}{2}\phi\cdot R\cdot\phi)},\\
&= -\frac{i}{2}\left(-i\frac{\delta}{\delta J}\right)\cdot\partial_k R\cdot\left(-i\frac{\delta}{\delta J}\right)e^{iW}\\
&= -\frac{i}{2}e^{iW}(\phi_c\cdot\partial_k R\cdot\phi_c)-\frac{1}{2}e^{iW}\mathrm{Tr}\left[(\partial_k R)\frac{\delta\phi_c}{\delta J}\right].
\end{aligned}
$$

- Hence we get $\partial_k W = -\frac{1}{2}(\phi_c\cdot\partial_k R\cdot\phi_c)+\frac{i}{2}\mathrm{Tr}\left[(\partial_k R)\frac{\delta\phi_c}{\delta J}\right]$.
- From this we find that the evolution of Γ is

$$
\partial_k\Gamma = \frac{i}{2}\mathrm{Tr}\left[(\partial_k R)\frac{\delta\phi_c}{\delta J}\right].
$$

ERG: The basics III

- From the definition of Γ we can get $J = -\left(\frac{\delta\Gamma}{\delta\phi_c} - R\cdot\phi_c\right)$,

ERG: The basics III

- From the definition of Γ we can get $J = -\left(\frac{\delta\Gamma}{\delta\phi_c} - R\cdot\phi_c\right)$,
- and hence $\frac{\delta J}{\delta\phi_c} = -(\Gamma^{(2)} - R)$,

ERG: The basics III

- From the definition of Γ we can get $J = -\left(\frac{\delta\Gamma}{\delta\phi_c} - R\cdot\phi_c\right)$,
- and hence $\frac{\delta J}{\delta\phi_c} = -(\Gamma^{(2)} - R)$,
- Here $\Gamma^{(2)} = \frac{\delta^2\Gamma}{\delta\phi_c\delta\phi_c}$.

ERG: The basics III

- From the definition of Γ we can get $J = -\left(\frac{\delta\Gamma}{\delta\phi_c} - R\cdot\phi_c\right)$,
- and hence $\frac{\delta J}{\delta\phi_c} = -(\Gamma^{(2)} - R)$,
- Here $\Gamma^{(2)} = \frac{\delta^2\Gamma}{\delta\phi_c\delta\phi_c}$.
- We can use this to express the evolution equation for Γ in the form of a one-loop integral,
  $\partial_k\Gamma = -\frac{i}{2}\mathrm{Tr}\left[(\partial_k R)\,(\Gamma^{(2)} - R)^{-1}\right]$.

ERG: The basics III

- From the definition of Γ we can get $J = -\left(\frac{\delta\Gamma}{\delta\phi_c} - R\cdot\phi_c\right),$
- and hence $\frac{\delta J}{\delta\phi_c} = -(\Gamma^{(2)} - R),$
- Here $\Gamma^{(2)} = \frac{\delta^2\Gamma}{\delta\phi_c\delta\phi_c}.$
- We can use this to express the evolution equation for Γ in the form of a one-loop integral,
  $\partial_k\Gamma = -\frac{i}{2}\mathrm{Tr}\left[(\partial_k R)\,(\Gamma^{(2)} - R)^{-1}\right].$
- Note that from now on we may drop the subscript c since the original quantum field does not appear in Γ.

Properties of R

- full effective action for $k \to 0$

Properties of R

- full effective action for $k \longrightarrow 0$
- classical action of FT as $k \longrightarrow \infty$

Properties of R

- full effective action for $k \to 0$
- classical action of FT as $k \to \infty$
- Choose k to add mass-gap for low-energy modes $(q < k)$

Properties of R

- full effective action for $k \longrightarrow 0$
- classical action of FT as $k \longrightarrow \infty$
- Choose k to add mass-gap for low-energy modes ($q < k$)
- $\partial_k R$ also UV regulator (bonus).

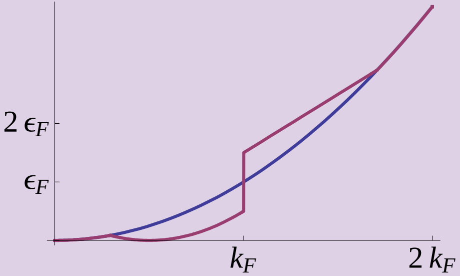
Properties of R

- full effective action for $k \longrightarrow 0$
- classical action of FT as $k \longrightarrow \infty$
- Choose k to add mass-gap for low-energy modes ($q < k$)
- $\partial_k R$ also UV regulator (bonus).

Properties of R

- full effective action for $k \to 0$
- classical action of FT as $k \to \infty$
- Choose k to add mass-gap for low-energy modes ($q < k$)
- $\partial_k R$ also UV regulator (bonus).

effect of R for fermions



The exact in ERG

- ERG is only exact if we can determine the functional $\Gamma[\phi]$

The exact in ERG

- ERG is only exact if we can determine the functional $\Gamma[\phi]$
- Where have we heard that before–on a par with Kadanov-Baym, Kohn-Sham, ...

The exact in ERG

- ERG is only exact if we can determine the functional $\Gamma[\phi]$
- Where have we heard that before–on a par with Kadanov-Baym, Kohn-Sham, ...
- Here the approximations will be done on the level of parametrising the functional.

The exact in ERG

- ERG is only exact if we can determine the functional $\Gamma[\phi]$
- Where have we heard that before–on a par with Kadanov-Baym, Kohn-Sham, ...
- Here the approximations will be done on the level of parametrising the functional.
- Richness of parametrisation links directly to quality of calculation

The exact in ERG

- ERG is only exact if we can determine the functional $\Gamma[\phi]$
- Where have we heard that before–on a par with Kadanov-Baym, Kohn-Sham, ...
- Here the approximations will be done on the level of parametrising the functional.
- Richness of parametrisation links directly to quality of calculation
- Guiding principle is gradient expansion

The exact in ERG

- ERG is only exact if we can determine the functional $\Gamma[\phi]$
- Where have we heard that before–on a par with Kadanov-Baym, Kohn-Sham, ...
- Here the approximations will be done on the level of parametrising the functional.
- Richness of parametrisation links directly to quality of calculation
- Guiding principle is gradient expansion
- Results will now depend on the choice of R

The exact in ERG

- ERG is only exact if we can determine the functional $\Gamma[\phi]$
- Where have we heard that before–on a par with Kadanov-Baym, Kohn-Sham, ...
- Here the approximations will be done on the level of parametrising the functional.
- Richness of parametrisation links directly to quality of calculation
- Guiding principle is gradient expansion
- Results will now depend on the choice of R
- There are some alternative approaches based on gridding fields, which have some limited use.

Attractive force for fermions: pairing

- Weak attractions: BCS
- Strong attraction: BEC

Model

- One type of fermion ψ (neutron matter or single species fermionic atomic gas)

Attractive force for fermions: pairing

- Weak attractions: BCS
- Strong attraction: BEC

Model

- One type of fermion ψ (neutron matter or single species fermionic atomic gas)
- Chemical potential μ

Attractive force for fermions: pairing

- Weak attractions: BCS
- Strong attraction: BEC

Model

- One type of fermion ψ (neutron matter or single species fermionic atomic gas)
- Chemical potential μ
- $\mathscr{A} = \int d^4x[\psi^\dagger(i\partial_t + \mu + \nabla^2/(2M))\psi + g(\psi^T\sigma_2\psi)(\psi^\dagger\sigma_2\psi^{\dagger T})]$

Attractive force for fermions: pairing

- Weak attractions: BCS
- Strong attraction: BEC

Model

- One type of fermion ψ (neutron matter or single species fermionic atomic gas)
- Chemical potential μ
- $\mathscr{A} = \int d^4x [\psi^\dagger (i\partial_t + \mu + \nabla^2/(2M))\psi + g(\psi^T \sigma_2 \psi)(\psi^\dagger \sigma_2 \psi^{\dagger T})]$
- Boson for correlated fermion pairs (and gap) ϕ through Hubbard-Stratonovich transformation.

Attractive force for fermions: pairing

- Weak attractions: BCS
- Strong attraction: BEC

Model

- One type of fermion ψ (neutron matter or single species fermionic atomic gas)
- Chemical potential μ
- $\mathscr{A} = \int d^4x [\psi^\dagger (i\partial_t + \mu + \nabla^2/(2M))\psi + g(\psi^T \sigma_2 \psi)(\psi^\dagger \sigma_2 \psi^{\dagger T})]$
- Boson for correlated fermion pairs (and gap) ϕ through Hubbard-Stratonovich transformation.
- $\phi^\dagger \to \psi^\dagger \sigma_2 \psi^{\dagger T}$

Effective action

$$
\begin{aligned}
\Gamma[\psi,\psi^\dagger,\phi,\phi^\dagger,\mu,k] \\
= \int d^4x \Bigg[& \phi^\dagger \left(Z_\phi (i\partial_t) + \frac{Z_m}{2m}\nabla^2 \right) \phi - U(\phi,\phi^\dagger) \\
& + \psi^\dagger \left(Z_\psi (i\partial_t + \mu) + \frac{Z_M}{2M}\nabla^2 \right) \psi \\
& - Z_g g \left(\frac{i}{2}(\psi^T \sigma_2 \psi)\phi^\dagger - \frac{i}{2}(\psi^\dagger \sigma_2 \psi^{\dagger T})\phi \right) \Bigg]
\end{aligned}
$$

- Bosons have become dynamical

Effective action

$$
\begin{aligned}
&\Gamma[\psi,\psi^\dagger,\phi,\phi^\dagger,\mu,k] \\
&= \int d^4x \Big[\phi^\dagger\left(Z_\phi(i\partial_t)+\frac{Z_m}{2m}\nabla^2\right)\phi - U(\phi,\phi^\dagger) \\
&\quad +\psi^\dagger\left(Z_\psi(i\partial_t+\mu)+\frac{Z_M}{2M}\nabla^2\right)\psi \\
&\quad -Z_g g\left(\frac{i}{2}(\psi^T\sigma_2\psi)\phi^\dagger - \frac{i}{2}(\psi^\dagger\sigma_2\psi^{\dagger T})\phi\right)\Big]
\end{aligned}
$$

- Bosons have become dynamical
- U contains 2μ term

Effective action

$$
\begin{aligned}
&\Gamma[\psi,\psi^\dagger,\phi,\phi^\dagger,\mu,k] \\
&= \int d^4x \Big[\phi^\dagger\left(Z_\phi(i\partial_t)+\frac{Z_m}{2m}\nabla^2\right)\phi - U(\phi,\phi^\dagger) \\
&\quad +\psi^\dagger\left(Z_\psi(i\partial_t+\mu)+\frac{Z_M}{2M}\nabla^2\right)\psi \\
&\quad -Z_g g\left(\frac{i}{2}(\psi^T\sigma_2\psi)\phi^\dagger-\frac{i}{2}(\psi^\dagger\sigma_2\psi^{\dagger T})\phi\right)\Big]
\end{aligned}
$$

- Bosons have become dynamical
- U contains 2μ term
- Many running couplings!

Effective action

$$\begin{aligned}\Gamma[\psi,\psi^\dagger,\phi,\phi^\dagger,\mu,k] \\ = \int d^4x \Bigg[& \phi^\dagger\left(Z_\phi(i\partial_t)+\frac{Z_m}{2m}\nabla^2\right)\phi - U(\phi,\phi^\dagger) \\ & +\psi^\dagger\left(Z_\psi(i\partial_t+\mu)+\frac{Z_M}{2M}\nabla^2\right)\psi \\ & -Z_g g\left(\frac{i}{2}(\psi^T\sigma_2\psi)\phi^\dagger - \frac{i}{2}(\psi^\dagger\sigma_2\psi^{\dagger T})\phi\right)\Bigg]\end{aligned}$$

- Bosons have become dynamical
- U contains 2μ term
- Many running couplings!
- Expansion points: Bosons around $\mathbf{k}=0$, fermions around Fermi momentum/energy (when we have it) or around $\mathbf{k}=0$, $E=-\Delta$ when in BEC phase.

Effective action

$$\Gamma[\psi, \psi^\dagger, \phi, \phi^\dagger, \mu, k]$$
$$= \int d^4x \left[\phi^\dagger \left(Z_\phi (i\partial_t) + \frac{Z_m}{2m}\nabla^2 \right) \phi - U(\phi, \phi^\dagger) \right.$$
$$+ \psi^\dagger \left(Z_\psi (i\partial_t + \mu) + \frac{Z_M}{2M}\nabla^2 \right) \psi$$
$$\left. - Z_g g \left(\frac{i}{2}(\psi^T \sigma_2 \psi)\phi^\dagger - \frac{i}{2}(\psi^\dagger \sigma_2 \psi^{\dagger T})\phi \right) \right]$$

- Bosons have become dynamical
- U contains 2μ term
- Many running couplings!
- Expansion points: Bosons around $\mathbf{k} = 0$, fermions around Fermi momentum/energy (when we have it) or around $\mathbf{k} = 0$, $E = -\Delta$ when in BEC phase.
- Not sure about full rigour in BEC phase.

Boson potential

- Expand U about equilibrium in constant background $\rho_0 = \phi_c^\dagger \phi_c$: (Note $\rho_0 \propto \Delta^2$!)

$$U = u_0 + u_1(\phi^\dagger\phi - \rho_0) + \frac{1}{2}u_2(\phi^\dagger\phi - \rho_0)^2 + \ldots.$$

Boson potential

- Expand U about equilibrium in constant background $\rho_0 = \phi_c^\dagger \phi_c$: (Note $\rho_0 \propto \Delta^2$!)

$$U = u_0 + u_1(\phi^\dagger\phi - \rho_0) + \frac{1}{2}u_2(\phi^\dagger\phi - \rho_0)^2 + \ldots.$$

- Two phases: symmetric where minimum at $\rho_0 = 0$;

Boson potential

- Expand U about equilibrium in constant background $\rho_0 = \phi_c^\dagger \phi_c$: (Note $\rho_0 \propto \Delta^2$!)

$$U = u_0 + u_1(\phi^\dagger \phi - \rho_0) + \frac{1}{2} u_2 (\phi^\dagger \phi - \rho_0)^2 + \ldots.$$

- Two phases: symmetric where minimum at $\rho_0 = 0$;
- condensed phase where $u_1 = 0$ at $\rho_0 \neq 0$.

Boson potential

- Expand U about equilibrium in constant background $\rho_0 = \phi_c^\dagger \phi_c$: (Note $\rho_0 \propto \Delta^2$!)

$$U = u_0 + u_1(\phi^\dagger\phi - \rho_0) + \frac{1}{2}u_2(\phi^\dagger\phi - \rho_0)^2 + \ldots.$$

- Two phases: symmetric where minimum at $\rho_0 = 0$;
- condensed phase where $u_1 = 0$ at $\rho_0 \neq 0$.
- Work at fixed density rather than fixed μ (want to study BEC, where $\mu < 0$).

Boson potential

- Expand U about equilibrium in constant background $\rho_0 = \phi_c^\dagger \phi_c$: (Note $\rho_0 \propto \Delta^2$!)

$$U = u_0 + u_1(\phi^\dagger\phi - \rho_0) + \frac{1}{2}u_2(\phi^\dagger\phi - \rho_0)^2 + \ldots.$$

- Two phases: symmetric where minimum at $\rho_0 = 0$;
- condensed phase where $u_1 = 0$ at $\rho_0 \neq 0$.
- Work at fixed density rather than fixed μ (want to study BEC, where $\mu < 0$).
- Solve system of coupled ODE's

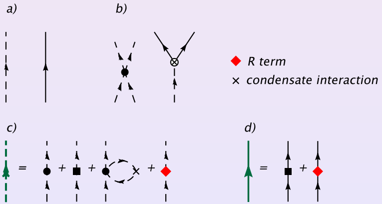
$$\Gamma^{(2)}_{BB} = \begin{pmatrix} \frac{\delta^2\Gamma}{\delta\phi^\dagger(-q')\delta\phi(q)} & \frac{\delta^2\Gamma}{\delta\phi^\dagger(-q')\delta\phi^\dagger(q)} \\ \frac{\delta^2\Gamma}{\delta\phi(-q')\delta\phi(q)} & \frac{\delta^2\Gamma}{\delta\phi(-q')\delta\phi^\dagger(q)} \end{pmatrix}$$

$$= \begin{pmatrix} Z_\phi q_0 - \frac{Z_m}{2m}\mathbf{q}^2 - u_1 - u_2(2\phi^\dagger\phi - \rho_0) & -u_2\phi\phi \\ -u_2\phi^\dagger\phi^\dagger & -Z_\phi q_0 - \frac{Z_m}{2m}\mathbf{q}^2 - u_1 - u_2(2\phi^\dagger\phi - \rho_0) \end{pmatrix}.$$

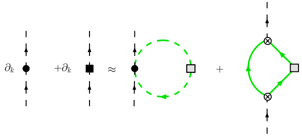
$$\Gamma^{(2)}_{FF} = \begin{pmatrix} \frac{\delta^2\Gamma}{\delta\psi(q')\delta\psi^\dagger(-q)} & \frac{\delta^2\Gamma}{\delta\psi^\dagger(q')\delta\psi^\dagger(-q)} \\ \frac{\delta^2\Gamma}{\delta\psi(q')\delta\psi(-q)} & \frac{\delta^2\Gamma}{\delta\psi^\dagger(q')\delta\psi(-q)} \end{pmatrix}$$

$$= \begin{pmatrix} Z_\psi q_0 - \frac{Z_M}{2M}(\mathbf{q}^2 - p_F^2) & ig\phi\sigma_2 \\ -ig\phi^\dagger\sigma_2 & Z_\psi q_0 + \frac{Z_M}{2M}(\mathbf{q}^2 - p_F^2) \end{pmatrix}.$$

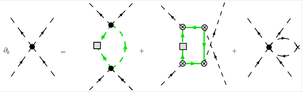
graphical representation



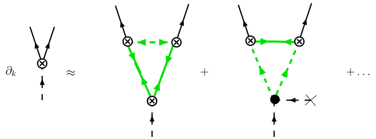
u_1:



u_2:



Z_g:



Initial Conditions

- Use standard approach to scattering theory (without pions)

Initial Conditions

- Use standard approach to scattering theory (without pions)
- Follow evolution from large $k = K$ to $k \approx 0$.

Initial Conditions

- Use standard approach to scattering theory (without pions)
- Follow evolution from large $k = K$ to $k \approx 0$.
- Initial conditions derived from matching to evolution in vacuum only.

Initial Conditions

- Use standard approach to scattering theory (without pions)
- Follow evolution from large $k = K$ to $k \approx 0$.
- Initial conditions derived from matching to evolution in vacuum only.
- Analytical solution for $\mu = 0$

$$u_1(K) = -\frac{M}{4\pi a_0} \int \frac{d^3q}{(2\pi)^3} \left[\frac{1}{E_{FR}(q,0,0)} - \frac{1}{E_{FR}(q,0,K)} \right]$$

Initial Conditions

- Use standard approach to scattering theory (without pions)
- Follow evolution from large $k = K$ to $k \approx 0$.
- Initial conditions derived from matching to evolution in vacuum only.
- Analytical solution for $\mu = 0$

$$u_1(K) = -\frac{M}{4\pi a_0} \int \frac{d^3q}{(2\pi)^3} \left[\frac{1}{E_{FR}(q,0,0)} - \frac{1}{E_{FR}(q,0,K)} \right]$$

- $u_1(0) = -\frac{M}{4\pi a_0}$ (equals T matrix)

Initial Conditions

- Use standard approach to scattering theory (without pions)
- Follow evolution from large $k = K$ to $k \approx 0$.
- Initial conditions derived from matching to evolution in vacuum only.
- Analytical solution for $\mu = 0$

$$u_1(K) = -\frac{M}{4\pi a_0}\int \frac{d^3q}{(2\pi)^3}\left[\frac{1}{E_{FR}(q,0,0)} - \frac{1}{E_{FR}(q,0,K)}\right]$$

- $u_1(0) = -\frac{M}{4\pi a_0}$ (equals T matrix)
- Difference of linearly divergent terms!

Mean Field

- Equations can be solved exactly in approximation where we neglect $\Gamma^{(2)}_{BB} - R$ term in running.

Mean Field

- Equations can be solved exactly in approximation where we neglect $\Gamma^{(2)}_{BB} - R$ term in running.
- Is just mean field theory.

Mean Field

- Equations can be solved exactly in approximation where we neglect $\Gamma^{(2)}_{BB} - R$ term in running.
- Is just mean field theory.
- Marani *et al* cond-mat/9703160, Papenbrock and Bertsch nucl-th/9811077.

Mean Field

- Equations can be solved exactly in approximation where we neglect $\Gamma^{(2)}_{BB} - R$ term in running.
- Is just mean field theory.
- Marani *et al* cond-mat/9703160, Papenbrock and Bertsch nucl-th/9811077.
- Finite T: Babaev

Mean Field

- Equations can be solved exactly in approximation where we neglect $\Gamma^{(2)}_{BB} - R$ term in running.
- Is just mean field theory.
- Marani *et al* cond-mat/9703160, Papenbrock and Bertsch nucl-th/9811077.
- Finite T: Babaev
- Agrees with numerics (next section)

Mean Field

- Equations can be solved exactly in approximation where we neglect $\Gamma^{(2)}_{BB} - R$ term in running.
- Is just mean field theory.
- Marani *et al* cond-mat/9703160, Papenbrock and Bertsch nucl-th/9811077.
- Finite T: Babaev
- Agrees with numerics (next section)
- But requires u_3: Use mean-field in full calculation.

Effective potential

Mean field effective potential ($k = 0$)

$$U^{MF}(\Delta, \mu) = \frac{k_\Delta^5}{2M\pi}\left[\frac{1}{8ak_\Delta} - \frac{1}{15}(1+x^2)^{3/4}P^1_{3/2}\left(-\frac{x}{\sqrt{1+x^2}}\right)\right]$$

$P^m_l(y)$ associated Legendre function; $k_\Delta = \sqrt{2M\Delta}$, $x = \mu/\Delta$.
Minimise w.r.t. Δ at fixed N, solve for μ and Δ.
log singularity at small Δ gives small $p_F a$ result

$$\Delta \approx \frac{8}{e^2}\varepsilon_F \exp\left(-\frac{\pi}{2p_F|a|}\right).$$

An example flow equation: U

With a uniform mean ϕ field (momentum conserved)

$$(\Gamma^{(2)}_{BB} - \mathbf{R}_B)^{-1} = \begin{pmatrix} Z_\phi q_0 - E_{BR}(q) + i\varepsilon & -u_2\phi\phi \\ -u_2\phi^\dagger\phi^\dagger & -Z_\phi q_0 - E_{BR}(q) + i\varepsilon \end{pmatrix}^{-1}$$

$$= \frac{1}{Z_\phi^2 q_0^2 - E_{BR}(q)^2 + V_B^2 + i\varepsilon} \begin{pmatrix} Z_\phi q_0 + E_{BR}(q) & -u_2\phi\phi \\ -u_2\phi^\dagger\phi^\dagger & -Z_\phi q_0 + E_{BR}(q) \end{pmatrix},$$

$E_{BR}(q) = \frac{Z_m}{2m} q^2 + u_1 + u_2(2\phi^\dagger\phi - \rho_0) + R_B(q,k)$, and
$V_B = u_2\phi^\dagger\phi$.

Multiplying by $\partial_k \mathbf{R}_B$ and trace gives

$$\frac{1}{2}\,\mathrm{tr}\left[(\partial_k \mathbf{R}_B)\,(\Gamma^{(2)}_{BB} - \mathbf{R}_B)^{-1}\right] = \frac{E_{BR}(q)\,\partial_k R_B(q,k)}{Z_\phi^2 q_0^2 - E_{BR}(q)^2 + V_B^2 + i\varepsilon}.$$

This has poles at $q_0 = \pm \frac{1}{Z_\phi}\sqrt{E_{BR}(q)^2 - V_B^2}$.

At $k = 0$ ($R_B = 0$) for $\phi^\dagger\phi \neq 0$, $u_1 = 0$)

$$q_0 = \pm \frac{1}{Z_\phi}\sqrt{\frac{Z_m}{2m}q^2\left(\frac{Z_m}{2m}q^2 + 2u_2\phi^\dagger\phi\right)},$$

and so the spectrum is gapless

Doing the q_0 integral (as a contour integral) gives

$$\int \frac{dq_0}{2\pi}\frac{1}{Z_\phi^2 q_0^2 - E_{BR}(q)^2 + V_B^2 + i\varepsilon} = -\frac{i}{2Z_\phi\sqrt{E_{BR}(q)^2 - V_B^2}}.$$

In a similar way, the fermion propagator is

$$\begin{aligned}(\Gamma^{(2)}_{FF} - \mathbf{R}_F)^{-1} &= \begin{pmatrix} Z_\psi q_0 - E_{FR}(q) + i\varepsilon\,\mathrm{sgn}(q-p_F) & ig\phi\sigma_2 \\ -ig\phi^\dagger\sigma_2 & Z_\psi q_0 + E_{FR}(q) - i\varepsilon\,\mathrm{sgn}(q-p_F) \end{pmatrix}^{-1} \\ &= \frac{1}{Z_\psi^2 q_0^2 - E_{FR}(q)^2 - \Delta^2 + i\varepsilon} \begin{pmatrix} Z_\psi q_0 + E_{FR}(q) & -ig\phi\sigma_2 \\ ig\phi^\dagger\sigma_2 & Z_\psi q_0 - E_{FR}(q) \end{pmatrix},\end{aligned}$$

where $E_{FR}(q) = \frac{Z_M}{2M}(q^2 - p_F^2) + R_F(q,p_F,k)\,\mathrm{sgn}(q-p_F)$, and $\Delta^2 = g^2\phi^\dagger\phi$.

Matrix trace:

$$\frac{1}{2}\,\mathrm{tr}\left[(\partial_k \mathbf{R}_F)\,(\Gamma^{(2)}_{FF} - \mathbf{R}_F)^{-1}\right] = \frac{2E_{FR}(q)\,\mathrm{sgn}(q-p_F)\,\partial_k R_F(q,p_F,k)}{Z_\psi^2 q_0^2 - E_{FR}(q)^2 - \Delta^2 + i\varepsilon}.$$

Poles at $q_0 = \pm\frac{1}{Z_\psi}\sqrt{E_{FR}(q)^2+\Delta^2}$.

For $k=0$ in the condensed phase:

$$q_0 = \pm\frac{1}{Z_\psi}\sqrt{\left(\frac{Z_M}{2M}(q^2-p_F^2)\right)^2+\Delta^2},$$

Gap at $q=p_F$ is $2\Delta/Z_\psi$.

Integrating over q_0 gives

$$\int\frac{dq_0}{2\pi}\,\frac{1}{Z_\psi^2 q_0^2 - E_{FR}(q)^2 - \Delta^2 + i\varepsilon} = -\frac{i}{2Z_\psi\sqrt{E_{FR}(q)^2+\Delta^2}}.$$

Full evolution equation for the potential

$$\partial_k U = -\frac{1}{\mathscr{V}_4}\partial_k\Gamma = \frac{1}{2Z_\phi}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{E_{BR}(\mathbf{q})}{\sqrt{E_{BR}(q)^2 - V_B^2}}\partial_k R_B(q,k)$$

$$-\frac{1}{Z_\psi}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{E_{FR}(q)}{\sqrt{E_{FR}(q)^2+\Delta^2}}\,\mathrm{sgn}(q-p_F)\,\partial_k R_F(q,p_F,k).$$

(Here $\mathscr{V}_4$ is the volume of spacetime.)

In the symmetric phase ($\rho = \rho_0 = 0$)

$$\partial_k u_1 = \frac{\partial}{\partial \rho}\left(\partial_k U\right)\Big|_{\rho=0} = \frac{g^2}{2Z_\psi}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{E_{FR}^2}\partial_k R_F,$$

$$\partial_k u_2 = \frac{\partial^2}{\partial \rho^2}\left(\partial_k U\right)\Big|_{\rho=0} = \frac{u_2^2}{2Z_\phi}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{E_{BR}^{(0)2}}\partial_k R_B$$

$$- \frac{3g^4}{4Z_\psi}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{E_{FR}^4}\partial_k R_F,$$

where $E_{BR}^{(0)}(q) = \frac{Z_m}{2m}q^2 + u_1 + R_B(q,k)$, and $E_{FR}(q)$ defined before.

In the condensed phase

$$\partial_k u_2 = \frac{\partial^2}{\partial\rho^2}\left(\partial_k U\right)\Big|_{\rho=\rho_0} = \frac{u_2^2}{2Z_\phi}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{\left(E_{BR}^{(v)}-2V_B^{(v)}\right)\left(E_{BR}^{(v)2}-6E_{BR}^{(v)}V_B^{(v)}+2V_B^{(v)2}\right)}{\left(E_{BR}^{(v)2}-V_B^{(v)2}\right)^{5/2}}\partial_k R_B$$

$$-\frac{3g^4}{4Z_\psi}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{E_{FR}}{\left(E_{FR}^2+\Delta^{(v)2}\right)^{5/2}}\,\mathrm{sgn}(q-p_F)\,\partial_k R_F,$$

where

$$\begin{aligned}
E_{BR}^{(v)}(q) &= \frac{Z_m}{2m}q^2+u_2\rho_0+R_B(q,k),\\
V_B^{(v)} &= u_2\rho_0,\\
\Delta^{(v)} &= g\sqrt{\rho_0}.
\end{aligned}$$

Flow over surface

In principle we don't have to solve at the minimum of potential. One can solve for the whole potential as e.g., a function of an input Δ and μ. The output then would be a set of parameters as a function of the input parameters. Find minimum of U, and thus determine equilibrium parameters.

Flow over surface

In principle we don't have to solve at the minimum of potential. One can solve for the whole potential as e.g., a function of an input Δ and μ. The output then would be a set of parameters as a function of the input parameters. Find minimum of U, and thus determine equilibrium parameters.

Alternative approach: Follow the minimum of the potential, keeping density fixed. This means we take a path through the infinite dimensional space where Δ and μ run.

Flow over surface

In principle we don't have to solve at the minimum of potential. One can solve for the whole potential as e.g., a function of an input Δ and μ. The output then would be a set of parameters as a function of the input parameters. Find minimum of U, and thus determine equilibrium parameters.
Alternative approach: Follow the minimum of the potential, keeping density fixed. This means we take a path through the infinite dimensional space where Δ and μ run.
Start in gap-less phase at large scale K, follow evolution down until u_1 hits zero, then gap starts to evolve, and we impose the condition that u_1 remains zero, which implies an (implicit) evolution of the gap.

Approach to numerics

- Solve ODEs (ignore running of all Z's but boson Z_ϕ).

Approach to numerics

- Solve ODEs (ignore running of all Z's but boson Z_ϕ).
- Crucial (and difficult point) to study evolution at constant density

Approach to numerics

- Solve ODEs (ignore running of all Z's but boson Z_ϕ).
- Crucial (and difficult point) to study evolution at constant density
- Start in symmetric phase; rather trivial (unphysical) transition to broken phase.

Approach to numerics

- Solve ODEs (ignore running of all Z's but boson Z_ϕ).
- Crucial (and difficult point) to study evolution at constant density
- Start in symmetric phase; rather trivial (unphysical) transition to broken phase.
- Studied various $R_{B,F}$'s!

Bosonic regulator

Carry out all energy integrals (0-th component) in closed form.

Bosonic regulator

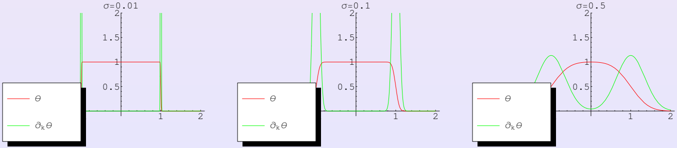
Carry out all energy integrals (0-th component) in closed form.Regulator only contributes to three-momentum integral:

$R_B = \frac{k^2}{2m} f(q/k)$ ($f(0) = 1, f(\infty) = 0$).

Use smoothed step function for f :

$f(x) = (\mathrm{erf}((x+1)/\sigma) + \mathrm{erf}((x-1)/\sigma))/(2\,\mathrm{erf}(1/\sigma))$

(Also $R_B = \frac{k^2}{2m} f(q)$)



Fermionic regulator

- Much more tricky;

Fermionic regulator

- Much more tricky;
- positive for particle state and negative for hole states since we do not work in a “sensible” particle-hole formalism, but relative to bare vacuum).

Fermionic regulator

- Much more tricky;
- positive for particle state and negative for hole states since we do not work in a “sensible” particle-hole formalism, but relative to bare vacuum).
- We use $R_F(q,k,p_F,\mu) = sgn(q-p_\mu)\frac{k^2}{2m}f((q-p_F)/k)$ $p_\mu = \sqrt{2M\mu}, p_F = (3\pi^2 n)^{1/3}$.

Fermionic regulator

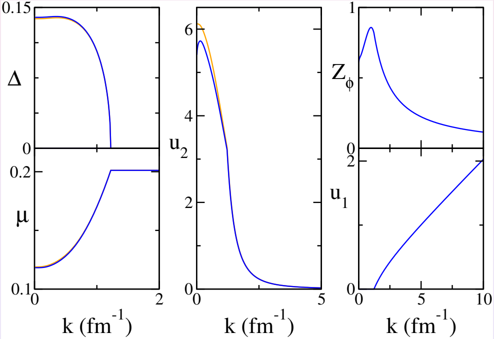
- Much more tricky;
- positive for particle state and negative for hole states since we do not work in a “sensible” particle-hole formalism, but relative to bare vacuum).
- We use $R_F(q,k,p_F,\mu) = sgn(q-p_\mu)\frac{k^2}{2m}f((q-p_F)/k)$ $p_\mu = \sqrt{2M\mu}, p_F = (3\pi^2 n)^{1/3}$.
- In absence of gap, $p_F = p_\mu$.

Fermionic regulator

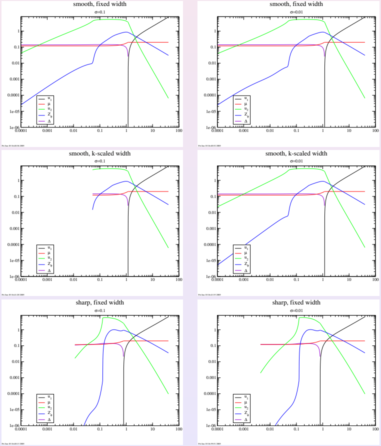
- Much more tricky;
- positive for particle state and negative for hole states since we do not work in a "sensible" particle-hole formalism, but relative to bare vacuum).
- We use $R_F(q,k,p_F,\mu) = sgn(q-p_\mu)\frac{k^2}{2m}f((q-p_F)/k)$ $p_\mu = \sqrt{2M\mu}, p_F = (3\pi^2 n)^{1/3}$.
- In absence of gap, $p_F = p_\mu$.
- With gap, Fermi surface will shift (to keep density constant), or disappear completely for BEC.

Fermionic regulator

- Much more tricky;
- positive for particle state and negative for hole states since we do not work in a "sensible" particle-hole formalism, but relative to bare vacuum).
- We use $R_F(q,k,p_F,\mu) = sgn(q-p_\mu)\frac{k^2}{2m}f((q-p_F)/k)$ $p_\mu = \sqrt{2M\mu}, p_F = (3\pi^2 n)^{1/3}$.
- In absence of gap, $p_F = p_\mu$.
- With gap, Fermi surface will shift (to keep density constant), or disappear completely for BEC.
- Use of p_F in f avoids complications with derivatives!

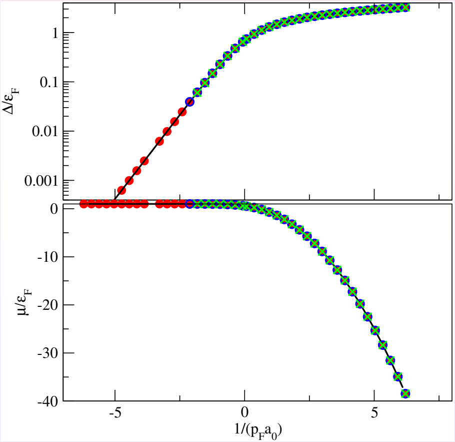


Numerical solution of the evolution equations for infinite a_0, starting from $K = 16\ \mathrm{fm}^{-1}$.
Blue: full solution, orange: “mean field”.
Transition to condensed phase at $k_{\mathrm{crit}} = 1.2\ \mathrm{fm}^{-1}$.
Contribution of boson loops small–tricky point!



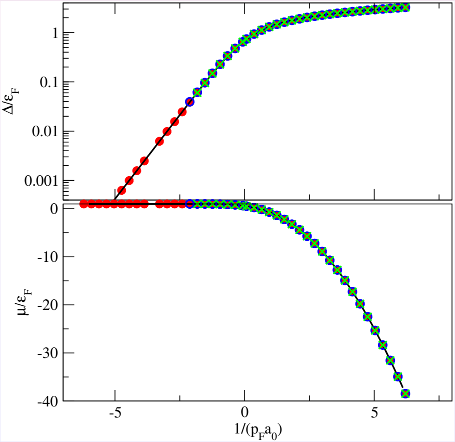
Cut-off dependence of solutions; log-log plots.

Crossover from BCS to BEC



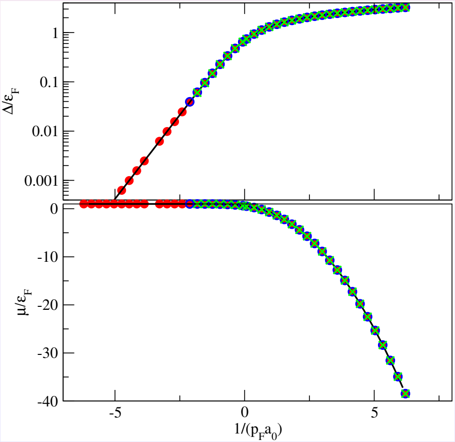
- p_F defined by density!

Crossover from BCS to BEC



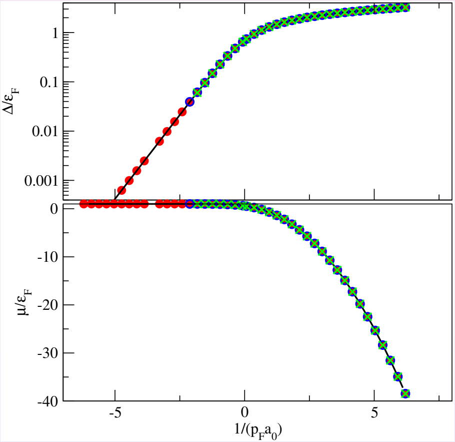
- p_F defined by density!
- find crossover from BCS to BEC

Crossover from BCS to BEC



- p_F defined by density!
- find crossover from BCS to BEC
- Little difference between mean field (red) and bosonic (green) results

Crossover from BCS to BEC



- p_F defined by density!
- find crossover from BCS to BEC
- Little difference between mean field (red) and bosonic (green) results
- problems with convergence in small gap regime

Tentative other results: Krippa

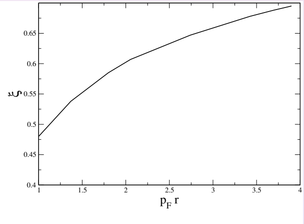


FIG. 1: The universal parameter ξ as the function of $p_F r$.

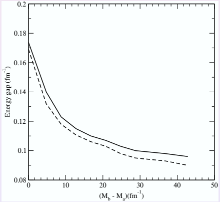


FIG. 1: Evolution of the gap in the MF approach (dashed curve) and with boson loops (solid curve) in the unitary regime $a = -\infty$ as a function of a mass asymmetry.

Wishlist

- Better understanding of role of Goldstone bosons.

Wishlist

- Better understanding of role of Goldstone bosons.
- Does our parametrization correspond to seperable pairing?

Wishlist

- Better understanding of role of Goldstone bosons.
- Does our parametrization correspond to seperable pairing?
- Complete analysis of Γ! (Wave function renormalisation constants, and coupling constants)

Wishlist

- Better understanding of role of Goldstone bosons.
- Does our parametrization correspond to seperable pairing?
- Complete analysis of Γ! (Wave function renormalisation constants, and coupling constants)
- Analytics mainly done, to be implemenented

Wishlist

- Better understanding of role of Goldstone bosons.
- Does our parametrization correspond to seperable pairing?
- Complete analysis of Γ! (Wave function renormalisation constants, and coupling constants)
- Analytics mainly done, to be implemenented
- Full inclusion of momentum dependent forces (effective range)

Wishlist

- Better understanding of role of Goldstone bosons.
- Does our parametrization correspond to seperable pairing?
- Complete analysis of Γ! (Wave function renormalisation constants, and coupling constants)
- Analytics mainly done, to be implemenented
- Full inclusion of momentum dependent forces (effective range)
- Treatment of *ph* channels.

Wishlist

- Better understanding of role of Goldstone bosons.
- Does our parametrization correspond to seperable pairing?
- Complete analysis of Γ! (Wave function renormalisation constants, and coupling constants)
- Analytics mainly done, to be implemenented
- Full inclusion of momentum dependent forces (effective range)
- Treatment of *ph* channels.
- Asymmetric matter

Wishlist

- Better understanding of role of Goldstone bosons.
- Does our parametrization correspond to seperable pairing?
- Complete analysis of Γ! (Wave function renormalisation constants, and coupling constants)
- Analytics mainly done, to be implemenented
- Full inclusion of momentum dependent forces (effective range)
- Treatment of *ph* channels.
- Asymmetric matter
- Three body forces (maybe)