

# Electron Correlations in Solids: From High-Temperature Superconductivity to Half-Metallic Ferromagnetism

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FWF projects n. P18505-N16, P18551-N16,  
DFG FOR 538

# Outline of the talk

- 1 Introduction: Correlation in High-Temperature Superconductors

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  - Nonquasiparticle states
  - CrO<sub>2</sub>
  - VAs: a correlation-induced half-metal ?

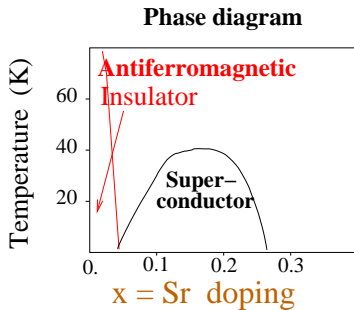
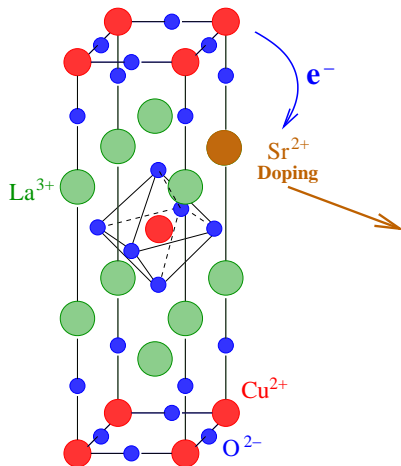
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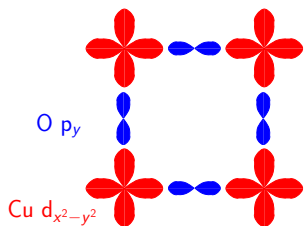
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# A High-Tc Superconductor

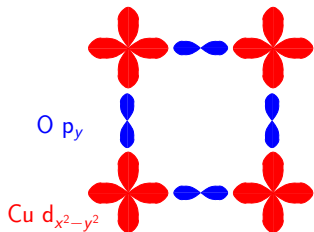




## Cu O<sub>2</sub> layer



# Cu O<sub>2</sub> layer

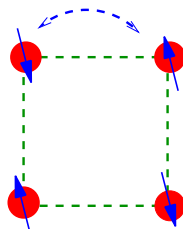


Reduced model

(e.g. Hubbard model)

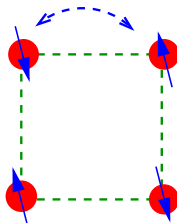


$t$   $\sim 0.5\text{eV}$   
Effective hopping strength



$$\underline{t} \sim 0.5 \text{ eV}$$

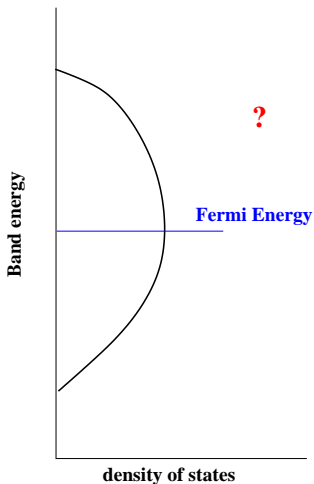
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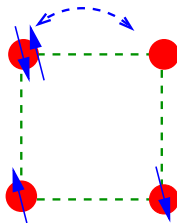
undoped compound

1 Electron per orbital

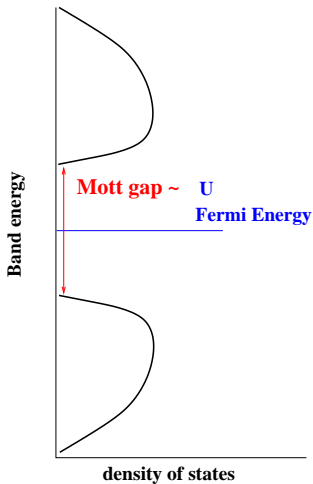
half filled band  $\rightarrow$  metal ??



$t \sim 0.5 \text{ eV}$   
effective hopping strength

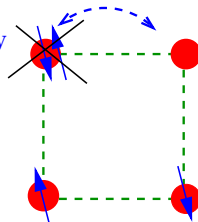


undoped compound  
1 Electron per orbital

**Mott insulator !**

Coulomb energy  $U \sim 4 \text{ eV}$   
 has to be paid !  
 double occupation  
 suppressed !

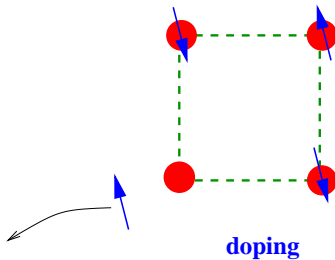
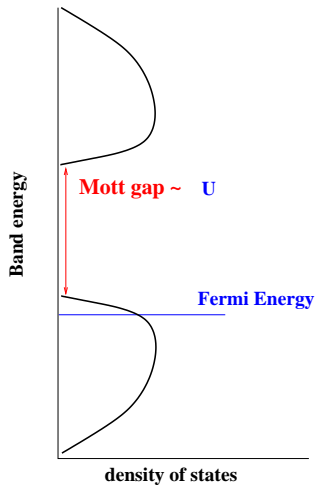
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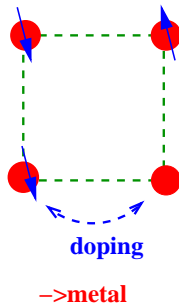
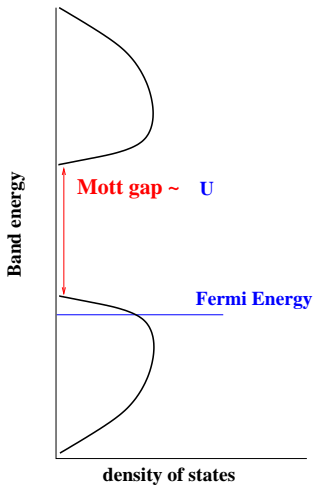


undoped compound

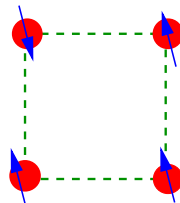
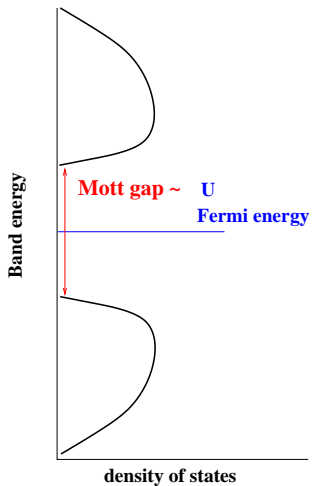
$\rightarrow$  insulator

correlation effects are important!





## magnetic properties



undoped compound

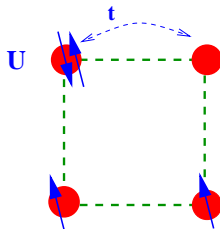


**magnetic properties**

kinetic energy is reduced  
due to

$$\Delta x \Delta p \geq \hbar$$

**virtual process**  
**Delocalisation**



## magnetic properties

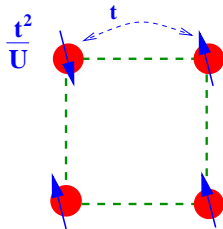
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$$\Delta E = -J \quad \text{for } (S_1 = -S_2)$$

$$J \sim t^2/U \sim 150 \text{ meV}$$

Superexchange energy

virtual process  
Delocalisation



**magnetic properties**

**not allowed !**

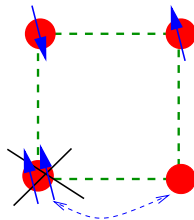
**due to Pauli principle**

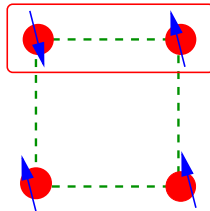
$$\Delta E = -J \quad \text{for}(S_1 = -S_2)$$

$$\Delta E = 0 \quad \text{for}(S_1 = S_2)$$

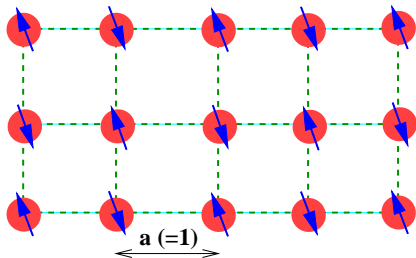
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**Superexchange energy**

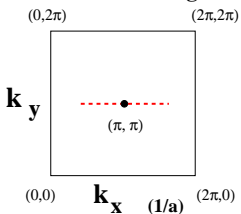


**magnetic properties****Superexchange prefers  
antiparallel spin configuration****Antiferromagnetism**

## Antiferromagnetism at $x=0$ (no doping)



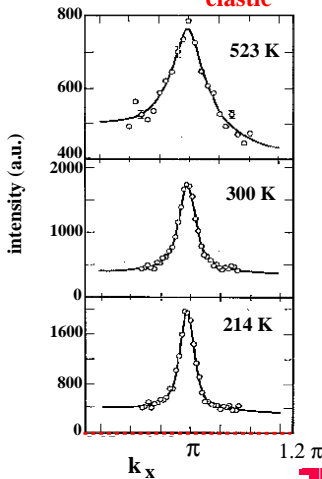
**neutron scattering:**



$\text{La}_2\text{CuO}_4$

Y. Endoh et al.  
PRB (88)

**elastic**



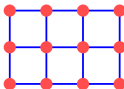
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## Appropriate numerical treatment of correlations?

1) "Exact" solution for a small cluster:

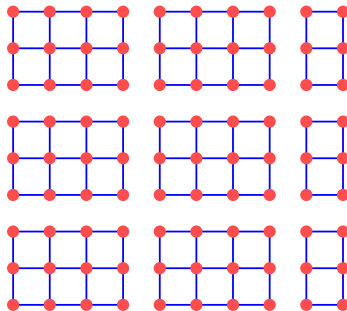
Quantum Monte Carlo  
Exact diagonalisation (Lanczos)



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Split infinite lattice  
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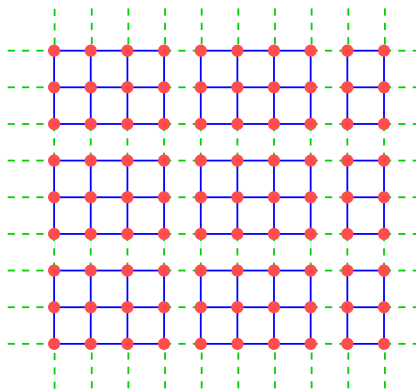
1) "Exact" solution for a small cluster:

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2) Perturbative treatment of intercluster hybridizations

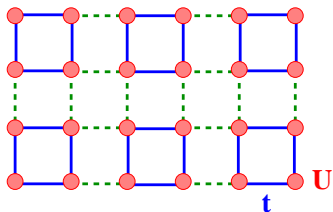
Cluster-perturbation theory (CPT)

(Gros, Valenti93; Senechal et al. 2000)



Split infinite lattice  
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## Cluster Perturbation Theory (CPT)



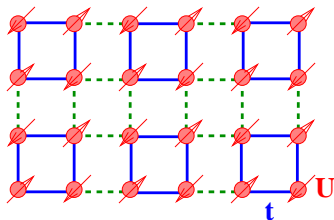
CPT:

$$\mathbf{H} = \mathbf{H}_{\text{cl}} + \mathbf{H}_{\text{intercl}}$$

$$\mathbf{G}_{\text{CPT}}^{-1} = \mathbf{G}_{\text{cl}}^{-1} - \mathbf{T}$$

(Gros, Valenti (93), Senechal et al. (00))

## Variational Cluster Approach (VCA)



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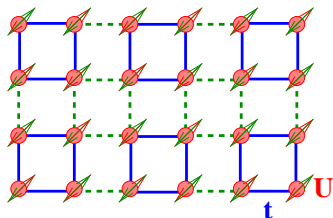
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Variational CPT : Treatment of symmetry-broken phases:

$$H'_{cl} = H_{cl} + h_{field}$$

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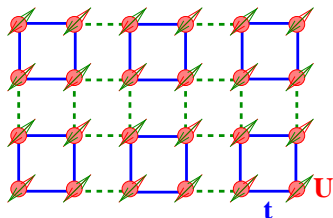
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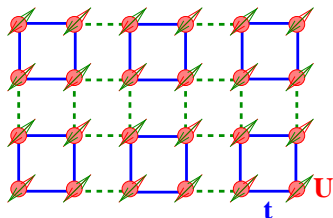
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How is  $h_{field}$  determined ?

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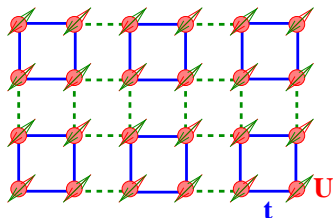
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**"Minimisation" of Grand-canonical (SFA) potential**

(Potthoff et al .03, Dahnken, Aichhorn, Hanke, Arrigoni, Potthoff 04)

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(Potthoff et al .03, Dahnken, Aichhorn, Hanke, Arrigoni, Potthoff 04)

Treatment of superconducting phase:

Senechal et al (05)

Aichhorn, Arrigoni(05)

Arrigoni, Aichhorn(06);

Aichhorn, Arrigoni, Potthoff, Hanke (06)

$$h_{SC} = \frac{\Delta}{2} \sum_{R,R'} \eta(R-R') (c_{R,\uparrow} c_{R',\downarrow} + h.c.),$$

# Selfenergy Functional Approach (SFA)

(M. Potthoff 2003)

- Many-Body Fermionic Hamiltonian:

$$H = H_0[G_0] \text{ (single-particle)} + U \text{ (interaction)}$$



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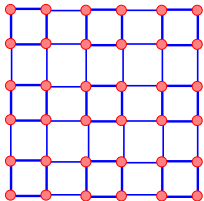
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# SFA — > Variational Cluster Approach

(M. Pottoff 2003) (C. Dahnken, M. Aichhorn, W. Hanke, E. Arrigoni, M. Potthoff 2004)

- Starting from  $H$

**H**



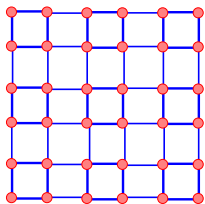


# SFA $\rightarrow$ Variational Cluster Approach

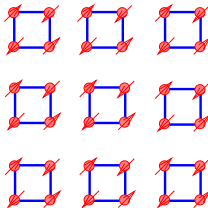
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**H**



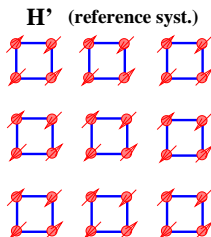
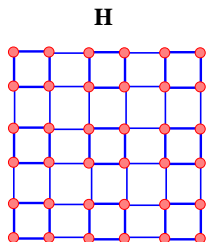
**H'** (reference syst.)



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- $$\Omega_{G_0}[\Sigma] = \Omega_{G'_0}[\Sigma] + \text{Tr} \ln(-(G_0^{-1} - \Sigma)^{-1}) - \text{Tr} \ln(-(G'_0{}^{-1} - \Sigma)^{-1})$$
- **Caveat**:  $F[\Sigma]$  can be evaluated only for a **restricted subspace of  $\Sigma$**   
 e.g. the ones that can be obtained from the cluster (cluster local)

# SFA – > Variational Cluster Approach

(M. Pottoff 2003) (C. Dahnken, M. Aichhorn, W. Hanke, E. Arrigoni, M. Pothhoff 2004)

- Consider a **Reference system**:  $H' = H_0[G'_0] + U$  (with the same  $U$ )
- $F[\Sigma]$  is the same for  $H'$  and  $H$  (universality)
- However**:  $\Omega_{G'_0}[\Sigma]$  (and thus  $F[\Sigma]$ ) can be evaluated exactly for  $H'$ :  
 $\Omega_{G'_0}[\Sigma] =$  grand-canonical potential
- The **exact**  $\Omega$  for  $H$  is then: (remember  $\Omega = F + \text{Tr} \ln(-(G_0^{-1} - \Sigma)^{-1})$ )

$$\Omega_{G_0}[\Sigma] = \Omega_{G'_0}[\Sigma] + \text{Tr} \ln(-(G_0^{-1} - \Sigma)^{-1}) - \text{Tr} \ln(-(G'_0{}^{-1} - \Sigma)^{-1})$$

- Caveat:  $F[\Sigma]$  can be evaluated only for a **restricted subspace** of  $\Sigma$
- The approximation consists in finding the **optimum**  $\Sigma$  within this subspace via  $\delta\Omega_{G_0}[\Sigma]/\delta\Sigma = 0$

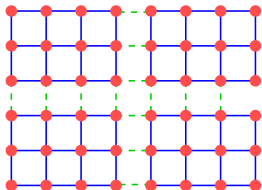
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  - This corresponds to the **optimisation** of the grand-canonical potential discussed before

## Model calculations

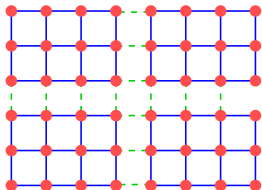
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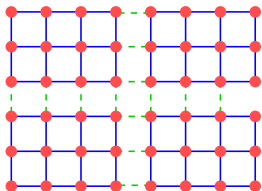


**Effort is concentrated**  
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phenomenological parameters  
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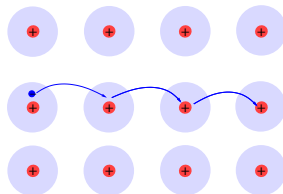
vs.

## Ab initio

Calculations within **density-functional theory (LDA,GGA,..)**

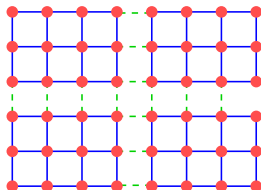
(W. Kohn, W. Kohn+ L. J. Sham ... Wien2k )

often very accurate  
start from "first principles"



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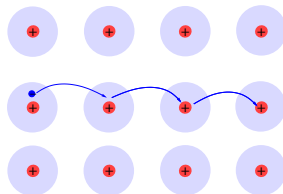
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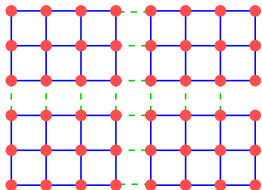


**However:**

Sometimes fail to explain some phenomena (High-T<sub>c</sub> superc., magnetism, Mott–insul.) in which correlations are important

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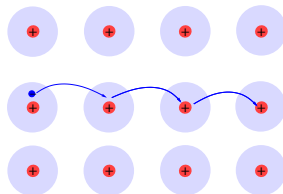
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**Can one combine the two ideas?**

yes ! **Combined approach:**

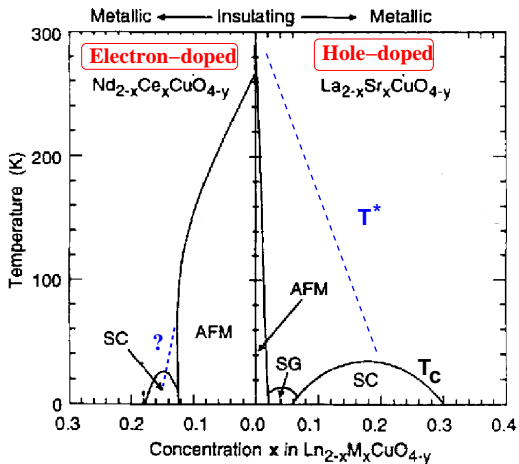
**LDA+ Dynamical Mean Field Theory**

(Anisimov et al., Kotliar+Vollhardt, Held, ...)

# Outline

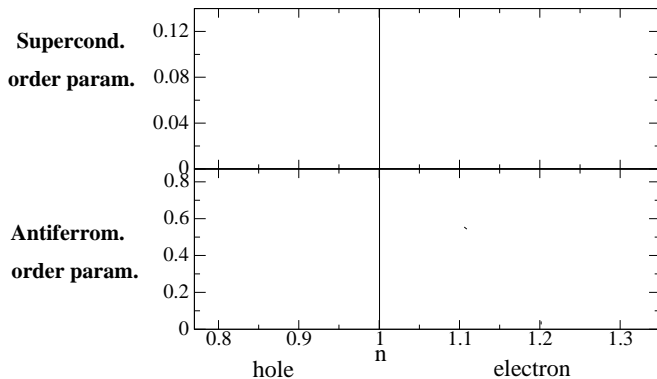
- 1 Introduction: Correlation in High-Temperature Superconductors
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# Electron and hole-doped High-T<sub>c</sub> Superconductors

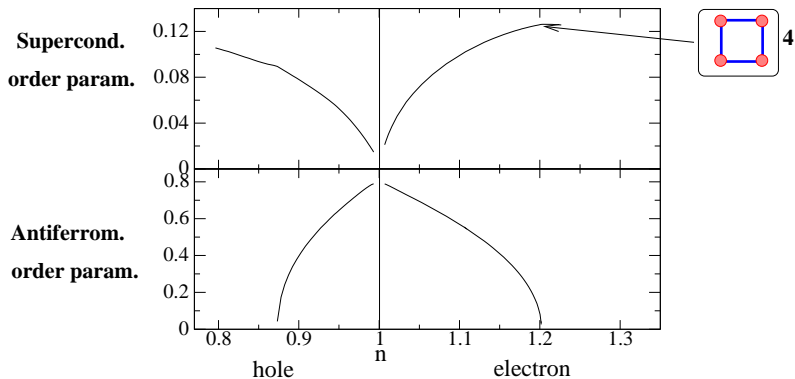


## Order Parameters

Single-band Hubbard model  $U/t=8$   $t'/t = -0.3$

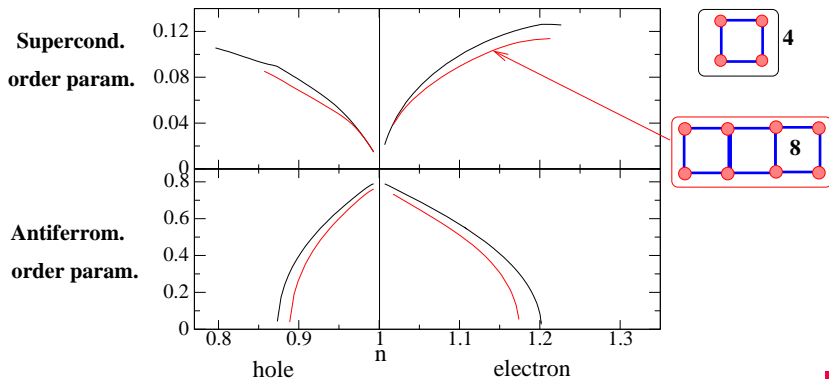


## Order Parameters



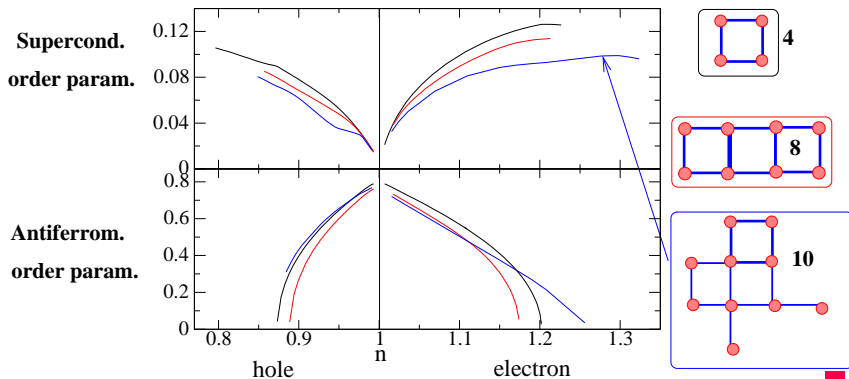


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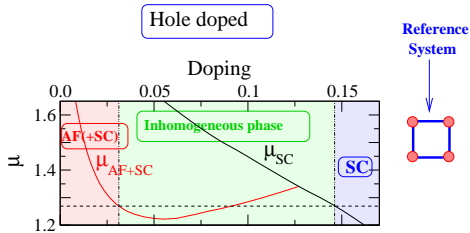


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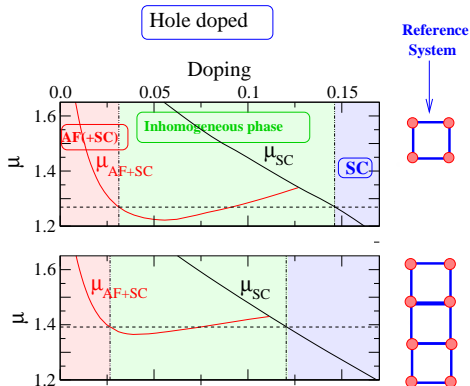
**Weak dependence on size of reference system (cluster)**



# Transition Antiferromagnetism – Superconductivity



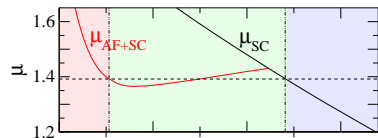
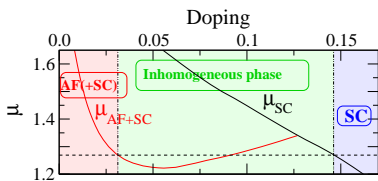
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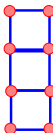
Aichhorn, Arrigoni, Hanke, Potthoff (2006)

# Transition Antiferromagnetism – Superconductivity

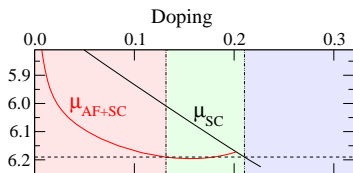
Hole doped



Reference System



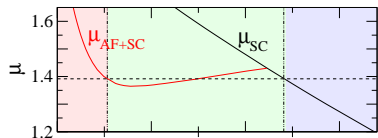
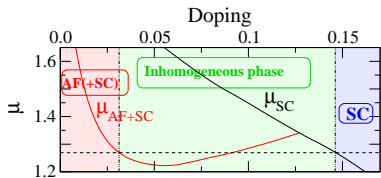
Electron doped



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# Transition Antiferromagnetism – Superconductivity

Hole doped

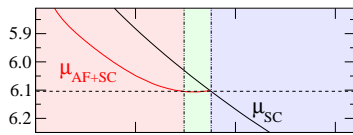
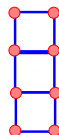
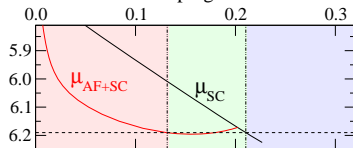


Reference System



Electron doped

Antiferromagnetic Phase is more stable



Aichhorn, Arrigoni, Hanke, Potthoff (2006)

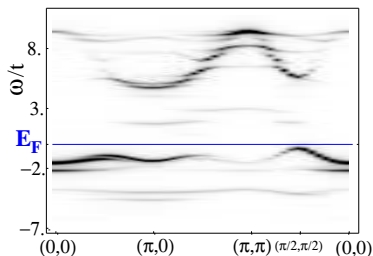
# Evolution of single-particle spectrum vs doping

ARPES=Angle-Resolved-Photoemission-Spectroscopy

(Aichhorn,Arrigoni EPL 2006)

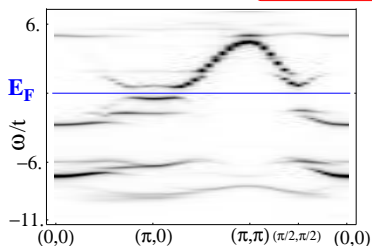
hole doped

$x=0.022$



electron doped

$x=0.13 \sim x_c$



holes first enter at  $(\pi/2, \pi/2)$

electron first enter at  $(\pi, 0)$

(In agreement with ARPES results for NdCeCuO

$(U/t=8, t'/t = -0.3)$

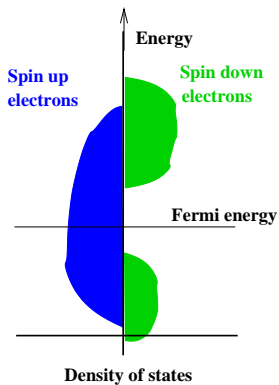
Armitage et al. 2001,2002)

# Outline

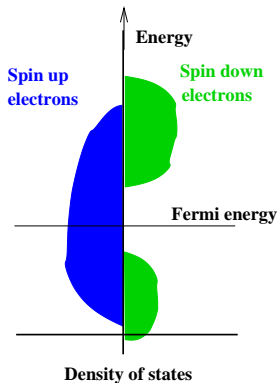
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## Half-Metallic Ferromagnets

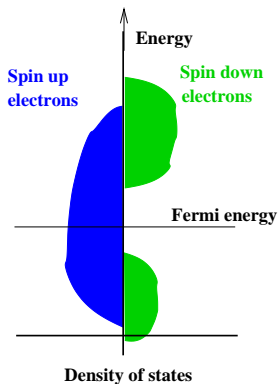


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Current is carried by  
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no contribution from spin down electrons!

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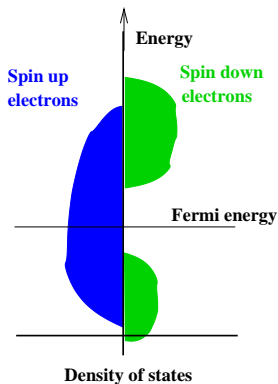
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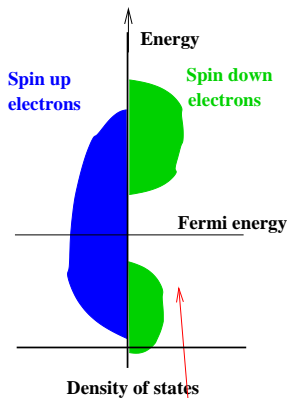
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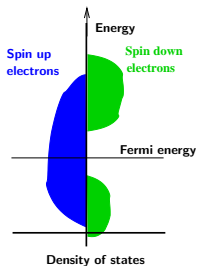
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However: these are results obtained by neglecting correlations!

## Correlation effects in half-metallic ferromagnets: formation of states within the gap

"non-quasiparticle states"

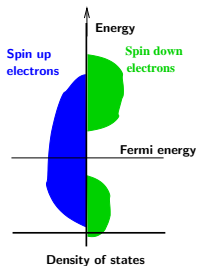
V. Yu. Irkhin and M. I. Katsnelson (90)  
D.M. Edwards and J. A. Hertz (73)



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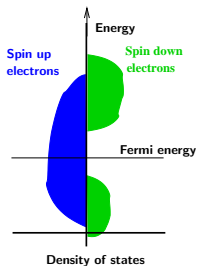
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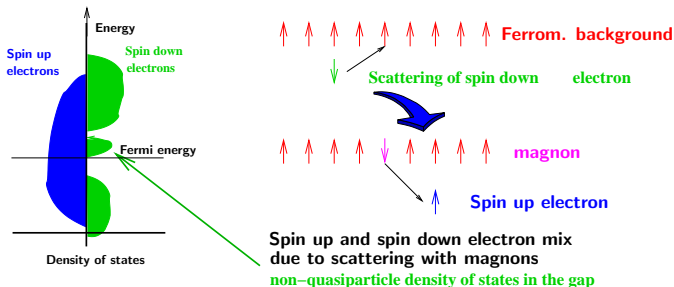




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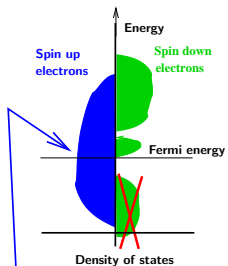
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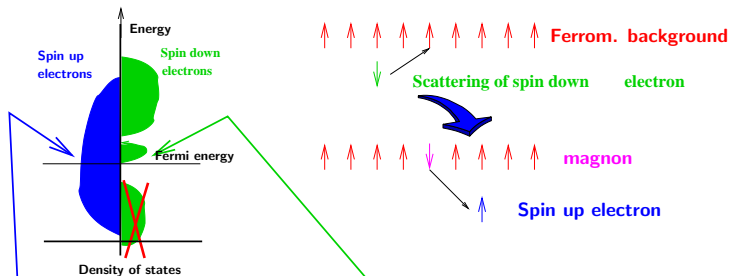


$(N_{\uparrow} = N + 1, N_{\downarrow} = 0)$  has spin:  $S = \frac{(N+1)}{2}, S_z = S$

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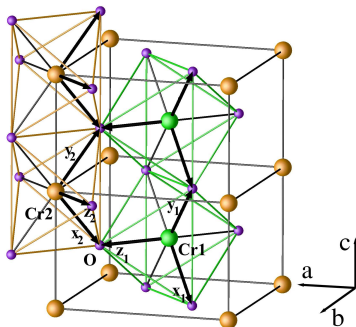
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State with  $S = \frac{(N+1)}{2}, S_z = S - 1,$

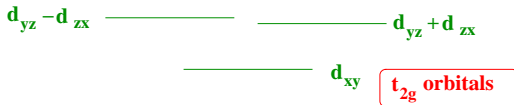
i. e.  $(N_{\uparrow} = N, N_{\downarrow} = 1)$  must have the same energy

CrO<sub>2</sub>

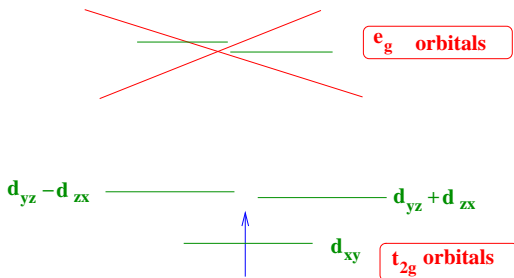
First predicted to be a half-metallic ferromagnet by [K.-H. Schwarz, J. Phys. F 19, L211 \(1986\)](#)

Optics, transport: [I. I. Mazin, D. J. Singh, and C. Ambrosch-Draxl, Phys. Rev. B \(1999\)](#)

... and many others ...

Building up the model: Relevant orbitals in CrO<sub>2</sub>

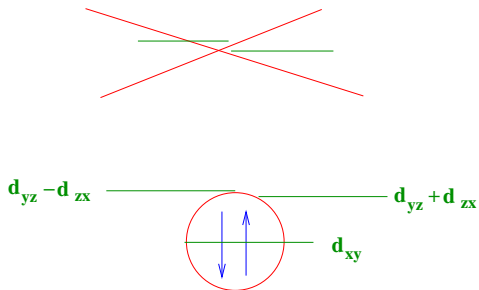
Cr 3d Orbitals,

Building up the model: Relevant orbitals in CrO<sub>2</sub>

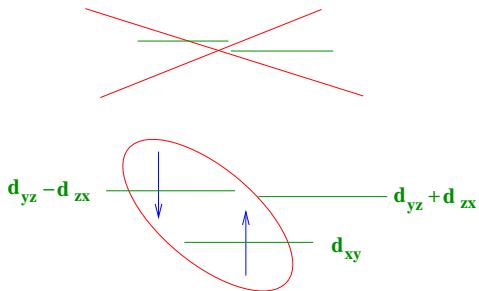
## Cr 3d Orbitals, crystal-field splitting

Effective model restricted to  $t_{2g}$  orbitals of Cr.  
 Parameters of the model obtained  
**AB INITIO** by downfolding of LDA bands  
 downfolding = integrating out high-energy bands

Andersen et al. (2000) ; Yamasaki et al. (2006)

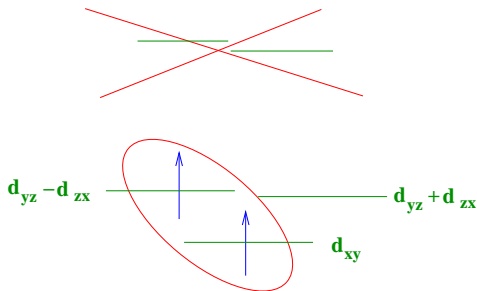
Building up the model: Relevant orbitals in  $\text{CrO}_2$ 

Interaction Energy  $U \approx 3\text{eV}$

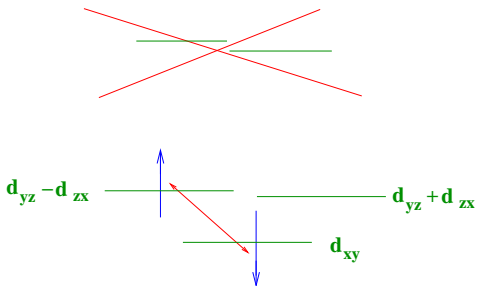
Building up the model: Relevant orbitals in CrO<sub>2</sub>

$$\text{Interaction Energy } U' = U - 2J$$

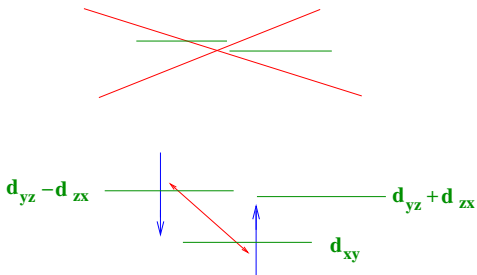


Building up the model: Relevant orbitals in CrO<sub>2</sub>

Interaction Energy  $U' - J$  (Hund's rule  $J \approx 0.9\text{eV}$ )

Building up the model: Relevant orbitals in CrO<sub>2</sub>

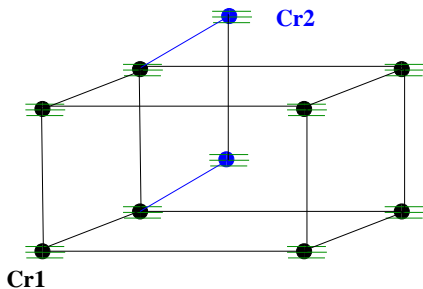
Spin-flip  $J$ : spin-rotation invariance

Building up the model: Relevant orbitals in  $\text{CrO}_2$ 

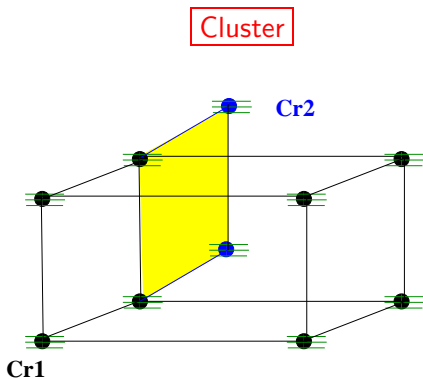
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# Model

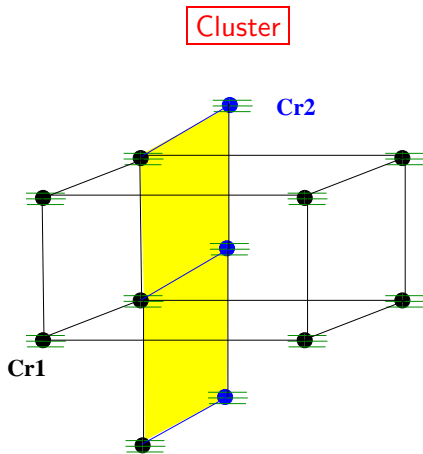
## Multi-orbital Hubbard model



# Model

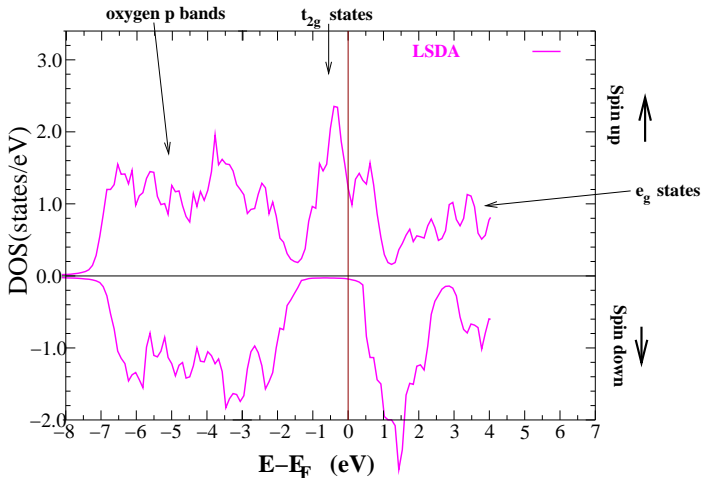


# Model



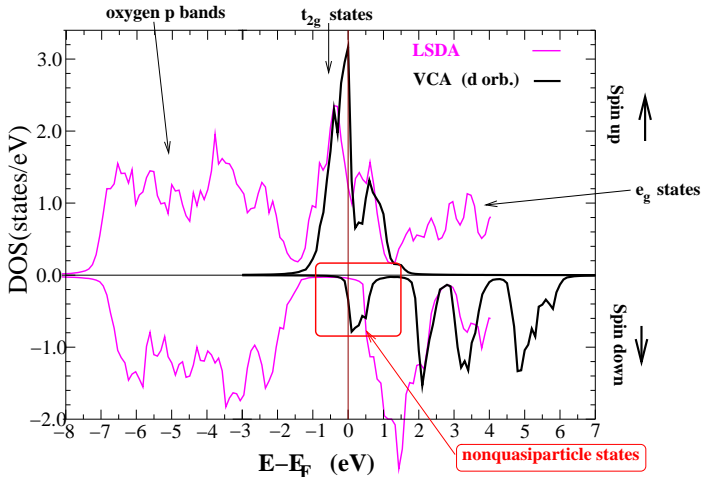
Results: Spin-resolved density of states for  $\text{CrO}_2$ :

## Spin-resolved LDA



Results: Spin-resolved density of states for CrO<sub>2</sub>:

VCA:Our calculation

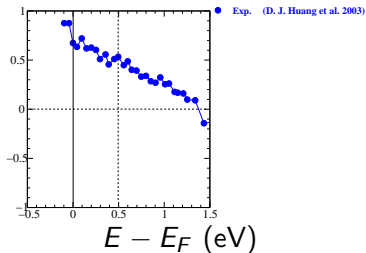




# Energy-Dependent Spin Polarisation

## Spin Polarisation

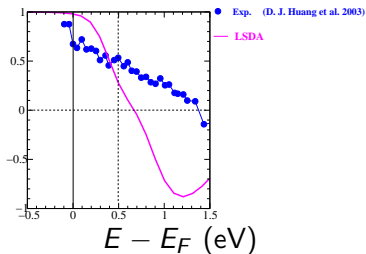
$$\frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$



# Energy-Dependent Spin Polarisation

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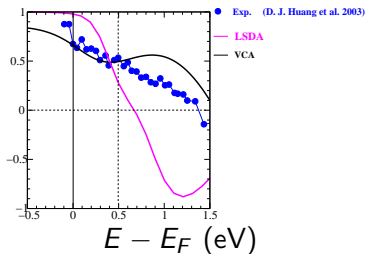
$$\frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$



# Energy-Dependent Spin Polarisation

## Spin Polarisation

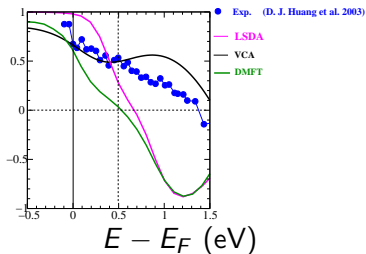
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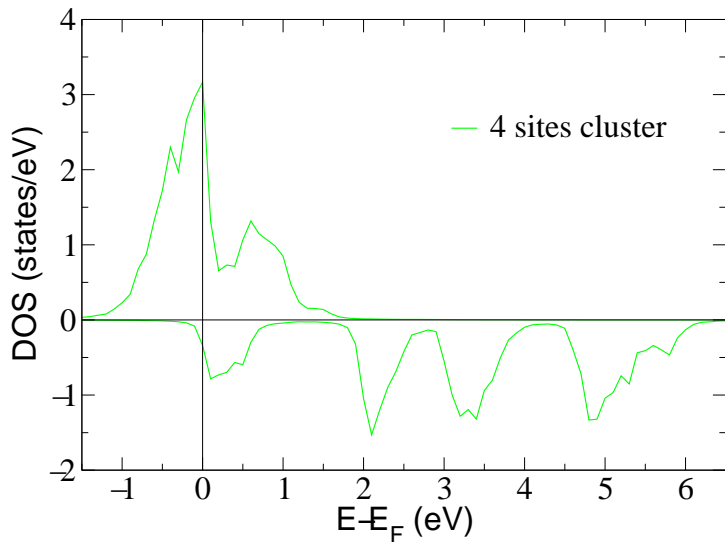
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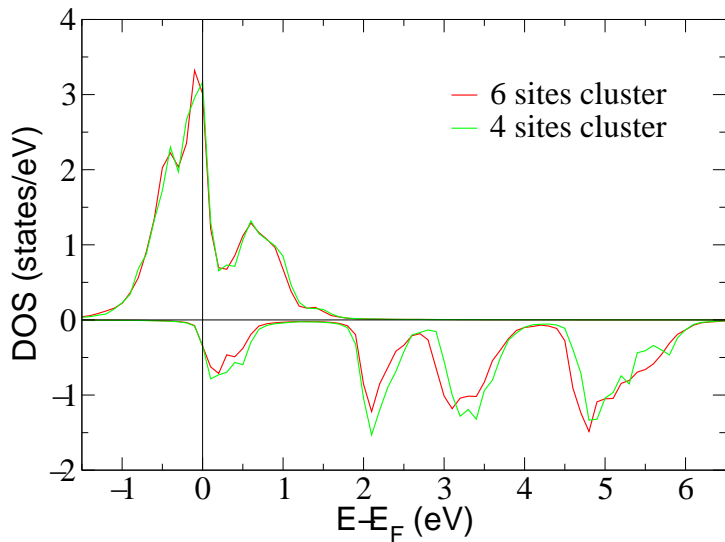


H. Allmaier et al. (Phys. Rev. B 2007)

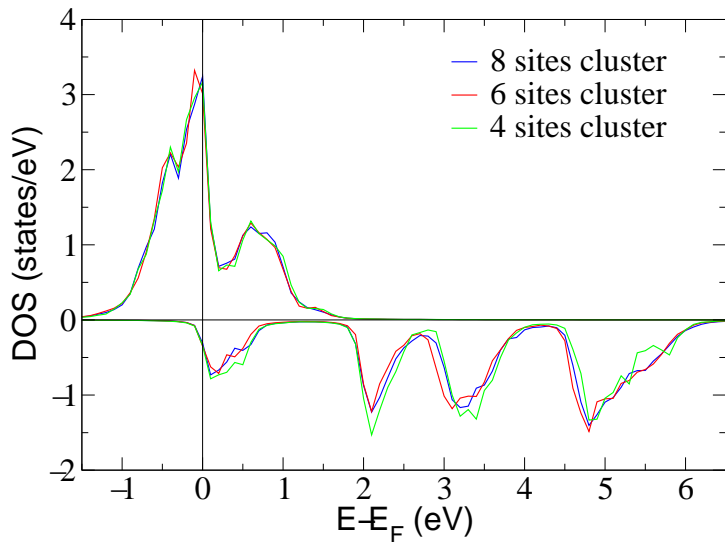
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# Electron correlation reduces polarisation



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Electron Correlation is bad for half metallicity?

## Correlation-induced half-metallicity in VAs?

( Chioncel, Mavropoulos, Lezaic, Blügel, Arrigoni, Katsnelson, Lichtenstein, PRL 2006)

LDA (GGA) calculations predict VAs (Zincblende) to be a  
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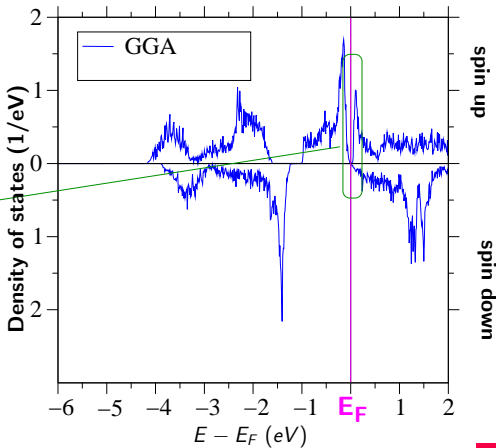
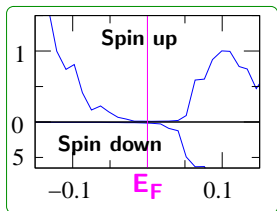
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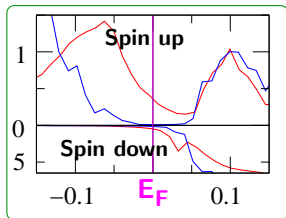
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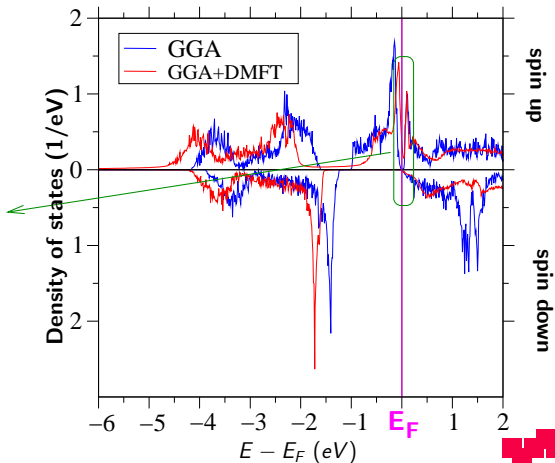
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Correlation effects  
fill the gap in spin up  
making VAs a  
half-metallic ferromagnet



# Outline

- 1 Introduction: Correlation in High-Temperature Superconductors
- 2 How do we deal with electron correlation?
  - Variational Cluster Approach (VCA)
  - Combination with *realistic ab initio* methods
- 3 Application: High-Temperature Superconductors: phase diagram
- 4 Application: Half-Metallic Ferromagnets
  - Nonquasiparticle states
  - CrO<sub>2</sub>
  - VAs: a correlation-induced half-metal ?
- 5 Summary and Outlook

# Thanks to

M. Aichhorn, M. Potthoff, W. Hanke (Würzburg)

L. Chioncel, M. Daghofer, H. Allmaier, A.-M. Fulterer (Graz)

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# FWF

FWF: P18551-N16 “Competing Phases in High-Temperature Superconductors: a theoretical investigation”

FWF P18505-N16 “Correlation effects in Half-Metallic ferromagnets”

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**DFG: FOR 538** “Doping dependence of phase transition and ordering phenomena in copper-oxide superconductors”



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- Correlation induced half-metallicity (VAs)
- Combination of **cluster calculations** (VCA) with **ab initio** methods (LDA,GGA)
- **Outlook:**
  - Full charge self consistency (VCA-LDA)
  - Surface effects (HMF)
  - VCA: Susceptibilities, DC conductivity